Selling Through Priceline? On the Impact of Name-Your-Own-Price in Competitive Market

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Priceline.com patented the innovative pricing strategy, Name-Your-Own-Price (NYOP), that sells opaque products through customer-driven pricing. In this paper, we study how competitive sellers with substitutable, non-replenishable goods may sell their products (1) as *regular* goods, through a direct channel at posted prices, and possibly at the same time (2) as *opaque* goods, through a third-party channel that engages in NYOP. We establish a stylized model framework that incorporates three sets of stakeholders: two competing sellers, an intermediary NYOP firm, and a sequence of customers. We first characterize customers' optimal purchasing/bidding decisions under various channel structures, and then analyze correspondin sellers' dynamic pricing equilibrium. We conduct extensive numerical studies to illustrate the impact of inventory and time on equilibrium prices, expected profit, and channel strategies. We find that the implications are highly dependent on channel structure (dual versus single). In particular, more inventory may reduce one's expected profit under dual structure, while this never happens when a seller uses direct channel only. Interestingly, although competing sellers seldom benefit from the existence of NYOP channel, it is possible that one or both of the sellers adopt it in equilibrium. We identify timing, inventory levels, and channel opaqueness as key drivers for NYOP adoption and characterize equilibrium areas for each type of channel structure.

Key words: Competition; Distribution channels; e-Commerce; Game Theory; Opaque Product; Pricing; Probabilistic Goods; Name-Your-Own-Price;

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1. Introduction

Decades since the emergence of e-commerce, online B2C shopping has entered into a renaissance age in which sellers actively seek innovative techniques that add flexibility to their offerings and operations. These innovations have taken place in: (1) the form of product/service offerings (e.g., opaque products offered by Hotwire.com, Priceline.com, and many tour operators), (2) the way prices are determined (e.g., customer-driven pricing at eBay.com and Priceline.com), and (3) the way in which transaction could be processed (e.g., quantity alarm set by Groupon.com). By releasing products, prices, and business transactions from their traditional, static roles, these innovations help ease capacity tension, refine market segmentation, and hedge for uncertainties in both supply and demand, which may ultimately enhance the efficiency of the system.

Apparently, one of the critical components in this process is the platform (e.g., Hotwire, Priceline, eBay, Groupon) that acts as the liaison between the product/service providers (e.g., hotels, manufactures) and the customers. In addition to complementing the traditional operations of the product/service providers, successful platforms can be quite rewarding to their hosting firms as well. For example, Priceline.com with its patented "Name-Your-Own-Price" (NYOP) mechanism has thrived ever since the dot-com bubble. Since the 2008 economic crisis, which created waves of price-conscious customers, it has overtaken its competitor Expedia.com in market capitalization (Tnooz.com, Nov 17, 2009), and then in gross bookings (Skift.com, Nov 8, 2013).

A mechanism that innovates on all three aforementioned dimensions (products, pricing, and processes), the NYOP business model has not yet been fully studied, especially in operations management literature. In this paper, we aim to deepen the understanding of NYOP from an operations perspective, and to offer insights to some fundamental questions.

To begin with, let us consider how NYOP works. Consider a customer who is looking for a round-trip air ticket from Los Angeles (LAX) to Montréal (YUL), departing on Feb 22 and returning on Feb 28. In acquiring such a trip via NYOP at Priceline, the customer needs to provide the departure and returning dates with respective airport codes, as well as the price that he/she is willing to pay. Priceline processes all the information and then notifies the customer whether his/her price is accepted. In particular, we would like to mention several distinctions of this process as follows:

• **Products**: apparently, there exists more than one flights between LAX and YUL on the designated dates, and Priceline can (if it will) assign the customer to any of them at its own discretion. That is to say, the customer will not learn the specifics of the trip (e.g., airlines, departure/arrival time, number of connections) until the transaction is finalized. In literature, such products whose characteristics are not fully revealed at the point of payment have been referred to as *opaque* goods (Anderson 2008, Fay 2008), as opposed to *regular* goods for which buyers know all features at the time of purchase.

• **Pricing:** different from the traditional *seller-driven* pricing, where a seller posts the price of the goods and a buyer simply makes a take-it-or-leave-it decision, in NYOP it is the buyer who offers a price and the seller determines whether to release a unit of the goods at that price. Therefore, NYOP belongs to the family of *buyer-driven* pricing. Both types of pricing mechanisms can be applied to either opaque or regular products. Table 1 summarizes how they have been implemented in industry.

• **Process**: NYOP is a commitment to buy. That is, once a customer's bid is accepted, he/she will be charged immediately, and the trip can no longer be altered (purchases through NYOP are

	Regular Goods	**Opaque Goods**
Seller-Driven	Expedia, Orbitz	Hotwire, BookIt
Buyer-Driven	Auction: eBay, Google NYOP: Unrevealed German NYOP firm (Hann and Terwiesch 2003, Terwiesch et al. 2005)	NYOP: Priceline Urbanoffer Germanwings

Table 1 Pricing Schemes for Regular and Opaque Goods

non-refundable and non-changeable). In case of a rejection, the transaction is cancelled. Technically, the customer cannot make the same bid within the next 24 hours[1].

Given these distinctions, it is unclear how NYOP may affect product/service providers' business. On one hand, it may help increasing market penetration (information-less opaque product can be sold at lower price), while on the other hand, it may cannibalize one's existing channel (customers may also find that buying at posted price could be quite costly since there is a chance to acquire the *same* item at lower price through NYOP). Sellers, therefore, have to balance the NYOP/opaque channel and posted-price/direct channel in a competitive market. These features motivate our interest to study strategic interactions among stakeholders in a supply chain setting. In particular, we investigate the impact of NYOP channel on market segmentation, pricing, and expected profit, as well as the main drivers for the adoption of the NYOP channel.

Our results provide many interesting implications to each stakeholder. On the market end, customers' channel and bidding strategies depend on the channel structure and their valuations of the products. For the competing sellers, equilibrium prices and profits can differ significantly under various channel structures. In particular, sellers' expected profits are generally lower when both adopt NYOP channel, and the expected profit can even be decreasing in the inventory levels, which never happens if at least one seller uses direct channel only. Interestingly, although the existence of NYOP channel seldom benefits the sellers, it is possible for both to adopt NYOP at the same time in equilibrium. In general, when NYOP product is opaque, the equilibrium channel structure can take many forms, and we identify areas for each of them in the numerical study. Finally, we show that for the intermediary NYOP firm, horizontally differentiated products are usually more beneficial than vertically differentiated ones. Thus, proper opaque product design, or supplier base construction, is paramount to the intermediary firm.

[1] Priceline may encourage customers to revise certain conditions *and* increase their prices in qualifying for a second bid. Yet the main tone is that one cannot repeatedly bid for the *same* item.

Our model adds a new block to the prior studies of NYOP: (1) We study dual channel–dynamic pricing strategy in a competitive market. Specifically, we allow the competing firms to adopt one or two channels throughout the time. The value of the NYOP channel is then analyzed in a more concrete manner; to the best of our knowledge, such issue only receive limited attention in relevant literature. (2) Different from many existing NYOP works, which are mainly customer-behavior oriented, our focus is more on the supply side. This emphasis bears practical meaning to industries where NYOP applies. We believe that these distinctions make the problem/model a relevant and interesting research topic.

The paper is organized as follows. We review the literature in §2 and introduce the model in §3. §4 studies customers' purchasing and bidding decisions under given prices and channel structures. §5 analyze the optimal or equilibrium pricing of the sellers. Extensive numerical study is conducted in §6, revealing important insights on the impact of inventory and time on pricing, profit, and channel decisions. §7 concludes and discusses some future extensions.

2. Literature Review

The business model of NYOP and opaque goods has drawn increasing attention in recent literature. In the early years, many papers on NYOP considered regular products only.[2] These papers are usually consumer-oriented, emphasizing consumer bidding behavior and/or restrictions that sellers may put on bidding (e.g., Hann and Terwiesch 2003, Fay 2004, Spann et al. 2004, Terwiesch et al. 2005, Hinz and Spann 2008, Fay and Laran 2009). However, most of the work assumes that NYOP is the only sales channel. Only recently the researchers started incorporating direct channels in the model. For example, Cai et al. (2009) study the value of double-bid option as well as direct channel with an NYOP seller. Wang et al. (2009) allow a firm to adopt NYOP only in last-minute sales, and find out that the value for an NYOP channel depends on the uncertainty of high-fare demand, rather than on the amount of excess capacity.

While most of the NYOP works consider single seller only, problems involving opaque goods usually demand a very different model structure (Granados et al. 2010), as it usually takes two or more sellers/products to provide an opaque offering. For this reason, competition may arise between firms contributing to the same opaque offering [3]. In addition, the opaque products can be priced in a seller-driven (posted price) or buyer-driven (e.g., auction, NYOP) manner. Fay (2008) models selling opaque products at posted prices only, and identifies conditions under which the

[2] Some of the models do not particularly differentiate regular versus opaque products—we put them under the column "Regular Goods," while the second column contains papers targeting only opaque products.

[3] There exists another line of research that considers opaque product being offered and controlled by one single firm. However, NYOP mechanism seldom applies in this case. We refer interested readers to Gallego and Phillips (2004), Jiang (2007), Fay and Xie (2008), Huang and Yu (2014), Zhang et al. (2015) and references therein.

opaque good may bring down the price in the traditional channel and harm the revenues. Jerath et al. (2010) consider a problem where firms can sell through a posted-price opaque channel in the last minute, and analyze respective benefit. Granados et al. (2015) examine the market response to opaque product and identify conditions under which it may cannibalize existing market or generate new demand. Granados et al. (2011) further provide evidence for the price-information trade-off within the opaque channel.

If we consider NYOP for pricing, the notion of "NYOP firm" needs further clarification. In the presence of opaque goods, the NYOP system usually consists of an NYOP intermediary firm and a number of sellers, in which the former acts as an intermediary that delegates the NYOP service for the latter. This differs from many aforementioned papers, in which the NYOP or opaque offerings are usually handled among the seller(s) themselves. In practice, however, the sellers and NYOP intermediary are usually separated: the sellers determine their reservation prices, and the NYOP intermediary selects the seller that accepts the lowest payment as the opaque product provider (Dolan 2000). In this area, Fay (2009) studies the competition between Priceline.com and Hotwire.com. Both companies deal with opaque products, but the former uses a buyer-driven NYOP scheme while the latter uses a seller-driven, posted-price scheme. Chen et al. (2014) consider heterogeneous customers and compare the performance of posted-price and NYOP in the last-minute opaque selling.

Among the existing works that study NYOP or opaque products, not much restriction has been put on the supply. The limited number of papers that do consider supply limit usually assumed that there can be only one distribution channel. For example, in Gallego and Phillips (2004), opaque products are available only during regular seasons, while in Wang et al. (2009), Jerath et al. (2010), and Chen et al. (2014) opaque selling occurs only in last minutes. However, it should be noted that in relevant industries where NYOP usually applies—e.g., airlines, hotels, and rental cars—supply/time limits play the pivotal role on revenue earning (McGill and van Ryzin 1999), and that these products/services are normally available in multiple sales channels (Bitran and Caldentey 2003, Elmaghraby and Keskinocak 2003). In approximating the real world practice, we consider a general model where competing sellers can dynamically determine their channel strategy, pricing strategy, and inventory control strategy during a finite time horizon. To our best knowledge, very few papers incorporated all these factors thus far.

3. The Model

Consider two sellers (each of whom will be referred to as "he") with substitutable products, an intermediary NYOP firm (referred to as "it"), and a sequence of customers (each of whom will be referred to as "she"). A seller and his product have the same index i, where $i \in \{1,2\}$. Assume that seller i holds $\bar{x}_i$ units of initial inventory, which will expire after T periods and is not replenishable. We use a backward time index $t = T, T-1, \ldots, 1$ to denote the current period, so a smaller number indicates that we are closer to the ending time. We refer to the time horizon from $t = T$ up to when the inventory is about to expire ($t = 1$) as the selling season of the products.

Prior to the selling season ($t = T$), sellers need to determine their channel strategies. Each seller owns a direct channel (e.g., company website) and contemplates whether to contract with the NYOP intermediary at the same time. Basically, a seller can choose to operate the direct channel without NYOP throughout the entire season; or, he can work with the NYOP intermediary and decide the usage of this channel in each period by adjusting his reservation price. Therefore, there could be three kinds of channel structures: 1) Single-Channel (SC), where both sellers use direct channels only, 2) Dual-Channel (DC), where both sellers sell through both the direct and the NYOP channels, and 3) Semi-Dual-Channel (SDCi), where seller $j = 3 - i$ uses direct channel only, and seller i adopts both the direct and the NYOP channels.

In each period $t \in \{T, T-1, ..., 1\}$, one customer arrives [4] and the same three-stage decision making takes place among the stakeholders, as depicted in Figure 1.

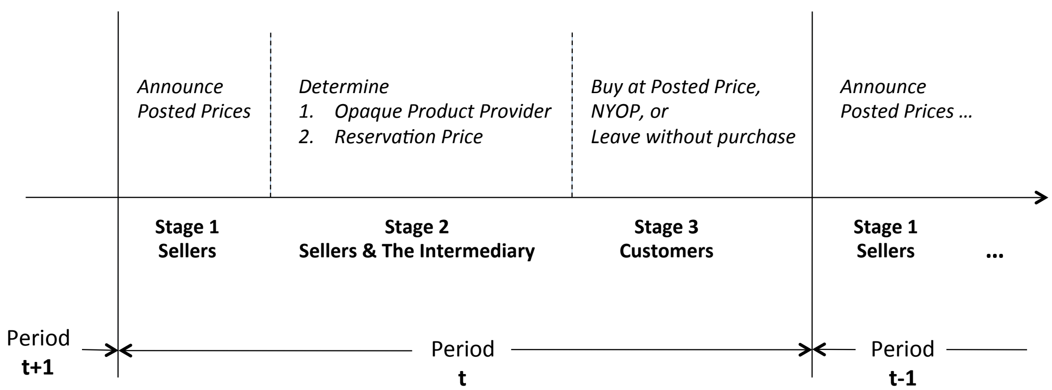

Figure 1 **Game sequence in each period.**

Stage 1: The sellers publish their posted prices, $\mathbf{p}_t = (p_{1t}, p_{2t})$, for purchases going through the direct channel during this period.

Stage 2: Each seller adopting the NYOP channel (e.g., seller i) sets a reservation price (e.g, r_{it}) with the intermediary firm. We model the intermediary firm as a revenue maximizer that benefits from the spread of NYOP bid and the reservation price (which aligns with the description in Dolan, 2000). That is, if the intermediary firm pairs an NYOP bid b with seller i's product, the

[4] This implicitly assumes that each period is rather short and corresponds to the inter-arrival time of the next customer. Similar assumptions have been widely applied in literature, e.g., Bhandari and Secomandi (2011), Kuo et al. (2011). This assumption is without loss of generality as it can be readily extended to fractional arrival rates (i.e., at most one customer arrives during a period) by adjusting the distributions of the valuations accordingly.

customer pays b, seller i receives r_{it}, and the intermediary earns the spread $b - r_{it}$. Then, under DC, the intermediary firm will find the lowest reservation price $r_t = \min \mathbf{r}_t = \min\{r_{1t}, r_{2t}\}$ and let seller $i = \arg\min_{k=1,2}\{r_{kt}\}$ be the opaque product provider. Specifically, under DC, we assume that the intermediary firm has the discretion to share r_t with the sellers in facilitating competitive reservation price submission. Under SDC, where only seller i participates in NYOP, the reservation price is $r_t = r_{it}$ and the opaque product provider is i.

Stage 3: A customer arrives with random valuation for each seller's product. The customer decides if she wants to buy directly from either seller at his posted price, or name her own price b_t (which we will refer to as "make a bid" for the reminder of the paper) with the intermediary firm. In the second case, she may be matched up with either of the two sellers if the bid is accepted. In case the bid is rejected, the customer can always buy from the direct channels. However, making a bid is considered a commitment to buy, and thus a customer cannot decline a product assigned by the intermediary firm if she discovers *ex-post* that being awarded the alternative product or buying at the posted price would make her better off. The customer is also not allowed to make a second bid with the NYOP channel after the first bid is rejected. These assumptions conform with the current practice of Priceline.com[5].

Assumptions: We assume that the customer has valuation $\mathbf{v}_t = (v_{1t}, v_{2t})$ for the products and $v_{i,t}$'s are independent and identically distributed (i.i.d.) on $[0, 1]$ with probability density function (p.d.f.) $f_i(\cdot)$ and cumulative distribution function (c.d.f.) $F_i(\cdot)$. The valuation is private information to the customer; the seller and the intermediary firm only know the distribution functions, $F_1(\cdot)$ and $F_2(\cdot)$. The customers are aware of the current posted prices $\mathbf{p}_t = (p_{1t}, p_{2t})$ and also whether NYOP is available, but have no access to the reservation price (r_t) or to the identity of i the opaque product provider.

Many NYOP papers (e.g., Hann and Terwiesch 2003, Spann et al. 2004, Terwiesch et al. 2005, and Cai et al. 2009) assume that consumers adopt subjective priors in estimating unobservable information such as reservation price. Similar assumptions also apply to opaque product realizations as opaque offerings are usually relegated to intermediaries with non-disclosure contractual clauses and there is a lack of creditable signalling in general (Fay 2008). This practically reflects customers' generic expectation of the NYOP/opaque channel due to lack of information, and could generally be approached by various marketing tools such as consumer survey. Conforming with this line of assumptions, we consider the customers assume that: 1) the opaque product provider can be either

[5] As Dolan (2000) noted, "...only one offer was permitted in a seven-day period..." The policy may have been updated, but Priceline technically forbids repeated bidding within a limited among of period. Interested readers may refer to Fay (2004), Spann (2004), Terwiesch et al. (2005), and Hinz and Spann (2008) for problems involving repeated bidding/bid learning.

seller 1 or seller 2 with probability α_1 and α_2, respectively, where $\alpha_1, \alpha_2 \geq 0$ and $\alpha_1 + \alpha_2 = 1$; and 2) the reservation price is a random variable $\hat{r}_t$ distributed on the support of $[0, \min\{p_{1t}, p_{2t}\}]$ with p.d.f. $g(\cdot)$ and c.d.f. $G(\cdot)$. Further, we assume that $\hat{r}_t$ has decreasing reversed hazard rate (DRHR), i.e., g/G is decreasing. This is equivalent to assuming that G is log-concave. Many (truncated) distributions satisfy this condition (e.g., uniform, normal, exponential, chi-squared, logit, etc.), hence this assumption is not too restrictive (Bagnoli and Bergstrom 2005).

Lastly, customers know about the availability status of a seller (e.g., a seller may show "in stock" or "stock out" in his direct channel) but not his real-time inventory level. For analytical convenience, we further assume that the inventory levels $\mathbf{x}_t = (x_{1t}, x_{2t})$ are visible between the sellers themselves. Similar assumptions were used in many papers regarding competitive revenue management (e.g., Gallego and Hu 2013; Levin et al. 2009; Lin and Sibdari 2008)[6]. As our primary goal is to study how the presence of an NYOP channel may affect the strategic decisions of sellers, this assumption falls within the scope of the paper.

4. The Customers' Problem

We first analyze the problem in Stage 3, where customers make purchasing/bidding decisions given the channel options. Since the same problem repeats itself every period, in this section we omit the subscript t for ease of exposition.

Denote a customer's action by $a \in A = \{0, 1, 2, B\}$. In particular, $a = 0$ suggests that a consumer would not purchase from any of the three channels (two direct channels and one opaque channel), and $a = 1$ if a consumer would buy directly from seller 1 at the posted price p_1. Similar interpretation applies to $a = 2$. If one chooses to NYOP first, then $a = B$ and the customer will have to pick an action from $A \backslash \{B\} = \{0, 1, 2\}$ if the bid is rejected.

We use $V_S^a(\mathbf{v})$ to represent the expected utility for a customer with valuation $\mathbf{v}$ taking action a under channel structure $S \in \{SC, SDC1, SDC2, DC\}$. For example, $V_S^0(\mathbf{v}) = 0$, $V_S^1(\mathbf{v}) = v_1 - p_1$ and $V_S^2(\mathbf{v}) = v_2 - p_2$ for any S. When $S = DC$, if a customer chooses to bid b first and then buy from seller 2 if rejected, her expected utility is $G(b)(\alpha_1 v_1 + \alpha_2 v_2 - b) + \bar{G}(b)(v_2 - p_2)$, where $\bar{G}(\cdot) = 1 - G(\cdot)$. A customer would rationally choose the bid and exit option (direct channel 1, or 2, or empty-handed) that maximizes her expected utility. Thus, $V_{DC}^B(\mathbf{v}) = \max_{b \geq 0}\{G(b)(\alpha_1 v_1 + \alpha_2 v_2 - b) + \bar{G}(b) \max_{k=0,1,2} V_{DC}^k(\mathbf{v})\}$. Similar analysis applies to SDCi.

Finally, in characterizing the final purchase, let $\mathbf{H} = (u, c)$ be a product-price tuple where $u \in \{0, 1, 2, O\}$ denotes the purchased product (no purchase, product 1, 2, or the opaque product respectively) and c is the price that one has to pay for the corresponding product.

[6] For imperfect information on parameters such as demand distributions or inventory levels, one may refer to Perakis and Sood (2006), Levina et al. (2009), Zhang and Kallesen (2008), etc.

4.1. SC: when neither seller adopts the NYOP channel

When both sellers decide not to participate in the opaque channel but operate the direct channel only, the action "B" is not available to the consumers. A customer's expected utilities under the rest of the actions are:

$$V_{SC}^{a}(\mathbf{v}) = \begin{cases} 0 & \text{if } a=0, \\ v_a - p_a & \text{if } a=1,2. \end{cases}$$

Comparing the expected utilities, the best action $a^*(\mathbf{v}) = \arg\max_{a\in\{0,1,2\}}\{V^a(\mathbf{v})\}$ is then

$$a_{SC}^{*}(\mathbf{v}) = \begin{cases} 0 & \text{if } \max\{v_1-p_1, v_2-p_2\} < 0; \\ 1 & \text{if } v_1-p_1 \geq \max\{v_2-p_2, 0\} \text{ and } p_1 \leq v_1; \\ 2 & \text{if } v_2-p_2 \geq \max\{v_1-p_1, 0\} \text{ and } p_2 \leq v_2; \end{cases}$$

and the final purchasing realization is

$$\mathbf{H}_{SC}(\mathbf{v}) = \begin{cases} (0,0) & \text{if } \max\{v_1-p_1, v_2-p_2\} < 0; \\ (1,p_1) & \text{if } v_1-p_1 \geq \max\{v_2-p_2, 0\}; \\ (2,p_2) & \text{if } v_2-p_2 \geq \max\{v_1-p_1, 0\}. \end{cases}$$

4.2. DC: When both sellers adopt the NYOP channel

In the case where both sellers sell through dual channels, the expected utility for a customer can be characterized as follows:

$$V_{DC}^{a}(\mathbf{v}) = \begin{cases} 0 & \text{for } a=0, \\ v_a - p_a & \text{for } a=1,2, \\ \max_{b\geq 0}\ G(b)(\alpha_1 v_1 + \alpha_2 v_2 - b) + \bar{G}(b)\max_{k=0,1,2}\{V_{DC}^{k}(\mathbf{v})\} & \text{for } a=B. \end{cases}$$

The best action for a customer with valuation $\mathbf{v}$ can be obtained as $a^*(\mathbf{v}) = \arg\max_{a\in\{0,1,2,B\}}\{V^a(\mathbf{v})\}$. In particular,

$$a_{DC}^{*}(\mathbf{v}) = \begin{cases} 1 & \text{if } v_1-p_1 \geq \max\{v_2-p_2, 0\} \text{ and } p_1/\alpha_2 < v_1-v_2; \\ 2 & \text{if } v_2-p_2 \geq \max\{v_1-p_1, 0\} \text{ and } p_2/\alpha_1 < v_2-v_1; \\ B & \text{if } \max\{v_1-p_1, v_2-p_2\} < 0; \\ & \text{or, } v_1-p_1 \geq \max\{v_2-p_2, 0\} \text{ and } p_1/\alpha_2 \geq v_1-v_2; \\ & \text{or, } v_2-p_2 \geq \max\{v_1-p_1, 0\} \text{ and } p_2/\alpha_1 \geq v_2-v_1. \end{cases}$$

That is, customers who can afford and strongly prefer product i to product j, where $i, j \in \{1,2\}$ and $i \neq j$, will buy product i directly at posted price p_i. The set of customers who will NYOP first contains three kinds of customers: budget customers, who can afford neither product at the posted prices, and customers who can afford and prefer product i (resp., j) to product j (resp., i) in a less-than-strong manner. Among these three kinds of customers, a rejected bid will lead to direct channel purchase for the latter two kinds of customers, and a budget customer will always leave empty handed if her bid is not accepted.

Proposition 1. *When both sellers sell through the opaque channel, 1) customers with limited degree of differentiation between the two products and 2) customers who can afford neither product at listed prices will NYOP first. In particular, the bid decreases with the degree of differentiation* ($|v_i - v_j + p_j - p_i|$) *for the first set of customers, and increases with the expected valuation* ($\alpha_i v_i + \alpha_j v_j$) *for the second set of customers.*

The optimal bid b^* for customers who will NYOP first ($a^* = B$) can be found from the first-order-condition on V^B: $b^* + \dfrac{G(b^*)}{g(b^*)} = \alpha_1 v_1 + \alpha_2 v_2 - \max\limits_{k=0,1,2}\{V^k(\mathbf{v})\}$. Since G/g increases in b, there exists a unique bidding level b^* that satisfies this condition. As $b + \frac{G(b)}{g(b)}$ increases in b, customers whose valuation $\mathbf{v}$ satisfies $r + \dfrac{G(r)}{g(r)} > \alpha_1 v_1 + \alpha_2 v_2 - \max\limits_{k=0,1,2}\{V^k(\mathbf{v})\}$ will bid lower than the reservation price ($b^*(\mathbf{v}) < r$), and hence be rejected in the NYOP channel. Consequently, the ultimate purchasing outcome can be characterized by

$$\mathbf{H}_{DC}(\mathbf{v}) = \begin{cases} (0,0) & \text{if } \max\{v_1 - p_1, v_2 - p_2\} < 0 \text{ and } r + \dfrac{G(r)}{g(r)} > \alpha_1 v_1 + \alpha_2 v_2; \\ (1, p_1) & \text{if } v_1 - p_1 \geq \max\{v_2 - p_2, 0\} \text{ and } p_1 - r - \dfrac{G(r)}{g(r)} < \alpha_2(v_1 - v_2); \\ (2, p_2) & \text{if } v_2 - p_2 \geq \max\{v_1 - p_1, 0\} \text{ and } p_2 - r - \dfrac{G(r)}{g(r)} < \alpha_1(v_2 - v_1); \\ (O, b^*) & \text{otherwise.} \end{cases}$$

Let us discuss the above result. First of all, budget customers with low expected valuation of the opaque product will bid low and will not be awarded any product. Next, customers with strong preference for product 1 over product 2 will buy directly instead of bidding, while the customers with moderate preference offer insufficient bids and turn back to direct channel 1 in the end; they are the customers who ultimately buy from direct channel 1. Similar analysis applies to direct channel 2. Lastly, customers who successfully obtained a product via NYOP consist of 1) budget customers with relatively high valuation of the two products, and 2) other customers who can afford one or both products at posted price, and have weak preference for one of the two direct channels.

4.3. SDC: When only seller i adopts the NYOP channel

When only one seller adopts the opaque channel (say, seller i sells through both channels but seller j's product is available at the direct channel only), the NYOP product is still available to customers. However, there are two different instances under which this case can occur.

In the first one, customers are not aware of sellers' specific channel decisions, hence the NYOP product is still opaque. This caters to the real life scenario in which partnership between the NYOP provider and the sellers is more pronounced than non-partner relationship. For example, opaque seller Hotwire.com may state that a 4-store hotel could possibly be "Westin, Sheraton or other brands and independents," but it will never make statement "Hyatt is excluded." In this instance, the same subjective prior (α_1, α_2) and $\hat{r}_t$ described for DC would be applied in estimating the opaque channel parameters. Consequently, customers' expected utility, V^a_{SDCi}, best action, a^*_{SDCi}, and ultimate purchasing outcome, $\mathbf{H}_{SDCi}$, take the same form as in DC.

In the second case, sellers' channel strategies are known to the customers, and the NYOP channel loses its opaqueness as customers are aware that only product i will be awarded. The customers essentially update their subjective priors to $\alpha_i = 1$ and $\alpha_j = 0$. The expected utility can then be expressed as:

$$V^a_{SDCi}(\mathbf{v}) = \begin{cases} 0 & \text{for } a = 0, \\ v_a - p_a & \text{for } a = 1, 2, \\ \max_{b \geq 0} \ G(b)(v_i - b) + \bar{G}(b) \max_{k=0,1,2}\{V^k_{SDCi}(\mathbf{v})\} & \text{for } a = B. \end{cases}$$

The best action $a^*(\mathbf{v}) = \arg\max_{a \in \{0,1,2,B\}}\{V^a(\mathbf{v})\}$ is

$$a^*_{SDCi}(\mathbf{v}) = \begin{cases} j & \text{if } v_j - p_j \geq \max\{v_i - p_i, 0\} \text{ and } p_j < v_j - v_i; \\ B & \text{otherwise.} \end{cases}$$

For customers who bid first ($a^* = B$), the optimal bid b^* satisfies $b^* + \dfrac{G(b^*)}{g(b^*)} = v_i - \max\limits_{k=0,1,2}\{V^k(\mathbf{v})\}$. Once again, since G/g increases in b, there exists a unique bidding level b^* that satisfies this condition. The final purchasing realization is then:

$$\mathbf{H}_{SDCi}(\mathbf{v}) = \begin{cases} (0,0) & \text{if } \max\{v_1 - p_1, v_2 - p_2\} < 0 \text{ and } r + \dfrac{G(r)}{g(r)} > v_i; \\ (i, p_i) & \text{if } v_i - p_i \geq \max\{v_j - p_j, 0\} \text{ and } p_i - r - \dfrac{G(r)}{g(r)} < 0; \\ (j, p_j) & \text{if } v_j - p_j \geq \max\{v_i - p_i, 0\} \text{ and } p_j - r - \dfrac{G(r)}{g(r)} < v_j - v_i; \\ (O, b^*) & \text{otherwise.} \end{cases} \tag{1}$$

Hereafter we refer to the first instance as *generic SDCi*, and to the second instance as *announced SDCi*.

5. Dynamic Pricing of the Sellers

This section provides structural results for the pricing decisions of two competing sellers (Stage 1 and Stage 2). The problem will be addressed in two steps. First, given posted prices $\mathbf{p}_t$ in period t, we investigate what reservation price r_t will the sellers chose and who will be the opaque provider. This corresponds to the Stage 2 decision. Second, we study whether equilibrium posted prices can be achieved—Stage 1 decision making.

5.1. Monopolistic seller

We first analyze the benchmark case in which there is only one seller, say, seller 1, in the market; this may happen when all inventories at seller 2 are sold out. Since availability is known to the customers, in this instance the NYOP channel loses its opaqueness and customers expect that only product 1 will be awarded for successful bids. We briefly state the key findings below. For tractability, we assume that the customers' valuation $\mathbf{v}$ are uniformly distributed on $[0,1]\times[0,1]$.

THEOREM 1. *A monopolistic seller maximizes his expected revenue by using the direct channel only.*

The result shows that the value of NYOP channel is minimum in absence of competition and opaqueness, as it does not provide the "shielding" benefit. In this circumstance, the NYOP channel allows customers to request a discount at little cost, as they can always buy from the direct channel guaranteeing the same product if their bid is turned down. Therefore, it is optimal for the customers to participate in the NYOP first, and only after that turn to the direct channel, if needed. Taking this into account, it is more profitable for the seller to sell everything via the direct channel. Specifically, we can characterize the optimal direct-channel pricing decision as follows:

$$p_{1t} = \begin{cases} \dfrac{1}{2} & \text{when } t=1; \\ \dfrac{1+\Pi_1(x_1,t-1)-\Pi_1(x_1-1,t-1)}{2} & \text{when } t>1, \end{cases}$$

where $\Pi_1(x_1,t)$ is the optimal expected profit for the monopolist seller in period t with x_1 units of inventory on hand:

$$\Pi_1(x,t) = \begin{cases} \dfrac{1}{4} & \text{when } t=1; \\ \Pi_1(x_1-1,t-1)+\left[\dfrac{1+\Pi_1(x_1,t-1)-\Pi_1(x_1-1,t-1)}{2}\right]^2 & \text{when } t>1. \end{cases}$$

5.2. SDC: Competing sellers, only one of whom is adopting the NYOP channel

This subsection considers equilibrium pricing under the SDC structure, where only one seller, e.g., seller 1, joins the NYOP channel throughout the selling season. Thus, he has to properly determine his posted and reservation prices, p_{1t} and r_{1t}, to control the sales flow.

Given any reservation and posted prices, $(\mathbf{r},\mathbf{p})$, let $\Omega_k(\mathbf{r},\mathbf{p})$, where $k=0,1,2,O$, be the fraction of customers who will *ultimately* buy nothing, product 1 from seller 1, product 2 from seller 2, or the opaque product from the intermediary firm, respectively. In other words, $\Omega_k(\mathbf{r},\mathbf{p}) = Prob\{\mathbf{H}_{SDC1} = k\}$. Then, at $(\mathbf{r}_t,\mathbf{p}_t)$,

$$\begin{aligned} \Pi_1(x_1,x_2,t) = \; & \Omega_O(\mathbf{r}_t,\mathbf{p}_t)\left[r_{1t}+\Pi_1(x_1-1,x_1,t-1)\right]+\Omega_1(\mathbf{r}_t,\mathbf{p}_t)\left[p_{1t}+\Pi_1(x_1-1,x_2,t-1)\right] \\ & +\Omega_2(\mathbf{r}_t,\mathbf{p}_t)\Pi_1(x_1,x_2-1,t-1)+\Omega_0(\mathbf{r}_t,\mathbf{p}_t)\Pi_1(x_1,x_2,t-1), \end{aligned}$$

$$\begin{aligned}\Pi_2(x_1,x_2,t) = \ & [\Omega_O(\mathbf{r}_t,\mathbf{p}_t)+\Omega_1(\mathbf{r}_t,\mathbf{p}_t)]\,\Pi_2(x_1-1,x_2,t-1) \\ & +\Omega_2(\mathbf{r}_t,\mathbf{p}_t)\,[p_{2t}+\Pi_2(x_1,x_2-1,t-1)]+\Omega_0(\mathbf{r}_t,\mathbf{p}_t)\Pi_2(x_1,x_2,t-1).\end{aligned}$$

Similar to §5.1, we assume that customers' valuation $\mathbf{v}$ is uniformly distributed on $[0,1]\times[0,1]$. Furthermore, we assume that customers adopt uniform distribution for the reservation price prior, i.e., $G(\cdot)\sim U[0,\min\{p_{1t},p_{2t}\}]$. As mentioned earlier, there are two scenarios to consider:

- In the case of *generic SDC*1, seller 1 choses (p_{1t}, r_{1t}) that maximizes $\Pi_1(x_1,x_2,t)$, and can possibly sell through both channels. This situation leads to a variety of possible outcomes which are difficult to characterize analytically; we discuss this case further in §6.4.2.
- Under *announced SDC*1, (1) in §4.3 suggests that for high reservation price (i.e., $2r_{1t}\geq p_{1t}$) no purchase will be done through the NYOP channel and all sales will be realized via direct channels ($\mathbf{H}(\mathbf{v})_{SDC1}\in\{1,2\}$); otherwise, customers will buy product 1 via the opaque channel and product 2 through the direct channel ($\mathbf{H}(\mathbf{v})_{SDC1}\in\{2,O\}$). Thus, under *announced SDCi*, seller i in each period can only sell either through the NYOP or through the direct channel. Then, given p_{jt}, the best pricing reaction for seller i is

$$(p^*_{it},r^*_{it}) = \begin{cases} (\arg\max_{0<p_{it}<1}\Pi_i^D,1) & \text{when } \Pi_i^D>\Pi_i^O; \\ (1,\arg\max_{0<r_{it}<1}\Pi_i^O) & \text{when } \Pi_i^D\leq\Pi_i^O, \end{cases}$$

where $\Pi_i^D=\Pi_i|_{r_{1t}=r_{2t}=1}$ and $\Pi_1^O=\Pi_1|_{p_{1t}=r_{2t}=1}$. Given (p_{it},r_{1it}), the optimal reaction for seller j can easily be presented as $p^*_{jt}=arg\max_{0<p_{jt}<1}\Pi_j|_{r_{jt}=1}$. It can then be proved that

Theorem 2. *Under announced SDCi, at any period t*

(i). seller i maximizes his expected revenue by using the direct channel only, i.e., $r^*_{it}=1$, $\forall t$*;*

(ii). there exists a pure-strategy Nash equilibrium in the posted prices (p^*_{1t},p^*_{2t}).

We can now compare results obtained for the semi-dual-channel with the monopoly case analyzed in §5.1. Our results for the SDCi case again confirm that NYOP is not suitable for environments that lack opaqueness. When customers are fully aware of the NYOP product origins, low reservation price can severely cannibalize sales from the direct channel, which makes NYOP unattractive to the seller. Therefore, when the channel structure is known to the customers, a sellers should not adopt NYOP alone and the announced SDC structure will ultimately be reduced to SC. This immediately leads to the following result:

Corollary 1. *Both SC and announced SCDi yield the same pricing decisions.*

5.3. DC: Competing sellers, both adopting the NYOP channel

We now study the DC scenario, in which both sellers adopt the NYOP channel, and opaqueness is preserved. To simplify our analysis, we define the *marginal value of inventory* at period t for each seller as

$$\widetilde{\Pi_1}(\mathbf{x},t) = \Pi_1(x_1, x_2 - 1, t) - \Pi_1(x_1 - 1, x_2, t),$$
$$\widetilde{\Pi_2}(\mathbf{x},t) = \Pi_2(x_1 - 1, x_2, t) - \Pi_2(x_1, x_2 - 1, t),$$

where $\widetilde{\Pi_i}$ measures the difference in seller i's expected revenue when he lets a customer go to his rival instead of serving the customer himself. In essence, it is the cost for seller i to serve a customer at period t and inventory level $\mathbf{x}$. Since the intermediary can disclose r_t to the sellers, it can be shown that in equilibrium the submitted reservation prices (r^*_{1t}, r^*_{2t}) reflect the marginal value of inventory for each seller, and the reservation price of the opaque product is the lower of the two marginal values.

For the reminder of the paper, denote by $\mathbf{p}^*(\mathbf{x}_t, t)$ the equilibrium posted price in period t when the inventory levels are $\mathbf{x}_t = (x_{1t}, x_{2t})$, and by $\Pi_i(\mathbf{x}_t, t)$ the expected revenue for seller i given that posted prices are set in equilibrium manner in all future periods. This leads to the following result.

PROPOSITION 2. *Under DC, at any period t,*

(i). the seller with lower marginal value of inventory will be the opaque product provider[7]*;*

(ii). the reservation price represents the higher marginal value of inventory between the two sellers (i.e., $r^*(\mathbf{x},t) = \max_{i=1,2} \widetilde{\Pi_i}(\mathbf{x}, t-1)$).

Given Stage 2 decision (reservation price and opaque product provider), we can now verify the existence of pure-strategy Nash equilibrium (NE), $(p^*_1(\mathbf{x},t), p^*_2(\mathbf{x},t))$, for Stage 1. In particular, we need to check if the expected profit function is quasi-concave in p_{it}. This can be analytically proved when customers' valuation $\mathbf{v}$ are uniformly distributed on $[0,1]\times[0,1]$ and have uniform distribution as their prior for reservation price, i.e., $G(\cdot) \sim U[0, \min\{p_{1t}, p_{2t}\}]$. Assuming these conditions are satisfied, the following result holds:

THEOREM 3. *Under DC, at any time epoch t, there exists a pure-strategy Nash equilibrium in posted prices $\mathbf{p}^*_t$.*

Although the proof of Theorem 3 is based on the aforementioned distributions,we conduct extensive numerical studies in §6 and find that the NE do exist under rather general settings, e.g., when $\mathbf{v}$ follows normal distribution. To a great extent, this shows the robustness of our model and justifies managerial insights derived afterwards.

Under the same set of assumptions that were used for Theorem 3, we can develop additional insights on the reservation price.

[7] In case of a tie, each serves as the opaque provider with an equal probability 0.5.

Corollary 2.

(i) In the last period, $t = 1$, the reservation price is zero if both sellers are in stock.

(ii) At period $t > 1$, reservation price r^* is zero if both sellers oversupply (i.e., $x_{i,t} \geq t$ for $i = 1, 2$).

The main implication of the above result is that sellers may offer last-minute sale ($t = 1$) that accepts arbitrary bids at the NYOP channel ($r_t^* = 0$); similar NYOP channel discounts can take place throughout the selling seasons when supply dominates demand. Note that in the latter case, deep discounts are mainly driven by competition and the existence of NYOP platform; in their absence, the discounts are less dramatic. In the monopolistic market, for example (§5.1), the seller will rather waste the excess capacity instead of lowering his posted price. In a competitive market without NYOP channel (as shown later in §6), oversupply may decrease the posted prices, but not to the extent in which customers pay the minimum reservation price for the product ($r_t^* = 0$).

The above analysis for different channel settings allows us to conduct numerical studies and obtain additional insights in the next section, focusing on expected profits, equilibrium prices, and equilibrium channel strategies.

6. Numerical Analysis: Profit, Pricing, and Channel Strategy

Our analysis thus far studied dynamic processes involved in obtaining equilibrium prices and profits under each channel structure. However, closed form solutions to these problems under general conditions are mostly intractable. In this section, we conduct numerical analysis and obtain additional insights to some more specific examples. Throughout the analysis, we apply the same customer valuation and prior as in §5 (i.e., $\mathbf{v} \sim U[0,1] \times [0,1]$, $\hat{r}_t \sim U[0, \min\{p_{1t}, p_{2t}\}]$). Unless specified otherwise, numerical results are based on $(\alpha_1, \alpha_2) = (0.5, 0.5)$. We further consider two sellers, each owning 8 units of items, and a selling season of 20 time epochs. We use dynamic programming to obtain the equilibrium prices and the expected profit at each possible inventory level and time epoch, (x_1, x_2, t), where $0 \leq x_1 \leq 8, 0 \leq x_2 \leq 8, 1 \leq t \leq 20$.

Throughout §6.1 to §6.3, we analyze how expected profit, equilibrium prices, and reservation price vary systematically with operational parameters (e.g., time, inventory levels, subjective priors, and channel structures). In this analysis, we mainly rely on SC and DC to illustrate the impacts of these parameters. In §6.4, we further allow the sellers to choose their own channel strategies and obtain the equilibrium channel structure, thus providing insight into some key drivers of NYOP adoption.

6.1. Expected Profit

Impact of Time. Intuitively, expected profit increases with leftover time for both SC and DC; we illustrate this in Figure 2. Specifically, we plot expected profit as a function of leftover time (t)

when one's own inventory level is fixed at $x_1 = 5$. We also allow the competitor's inventory level, x_2, to take a wide range of values from the set $\{0, 2, 5, 8\}$. Under SC, where neither of the sellers adopt NYOP, it can be observed that expected profit (Π_1) is increasing and concave in t; thus, the value of time has a diminishing rate of return for SC structure. The same insight holds for DC when the leftover time is sufficiently large (e.g., $t \geq 5$ when $x_2 = 5$, or $t \geq 8$ when $x_2 = 8$). However, the expected profit curve could be convex when t is small. Basically, when demand during the leftover time is not sufficient to support existing inventory at either seller ($t < x_1, x_2$), the sellers will engage in ferocious competition in both channels (opaque and direct), which severely deteriorates expected profits; this effect can be diminished with demand arrivals brought by additional time epochs. Thus, under DC, the value of time may also exhibit an increasing rate of return.

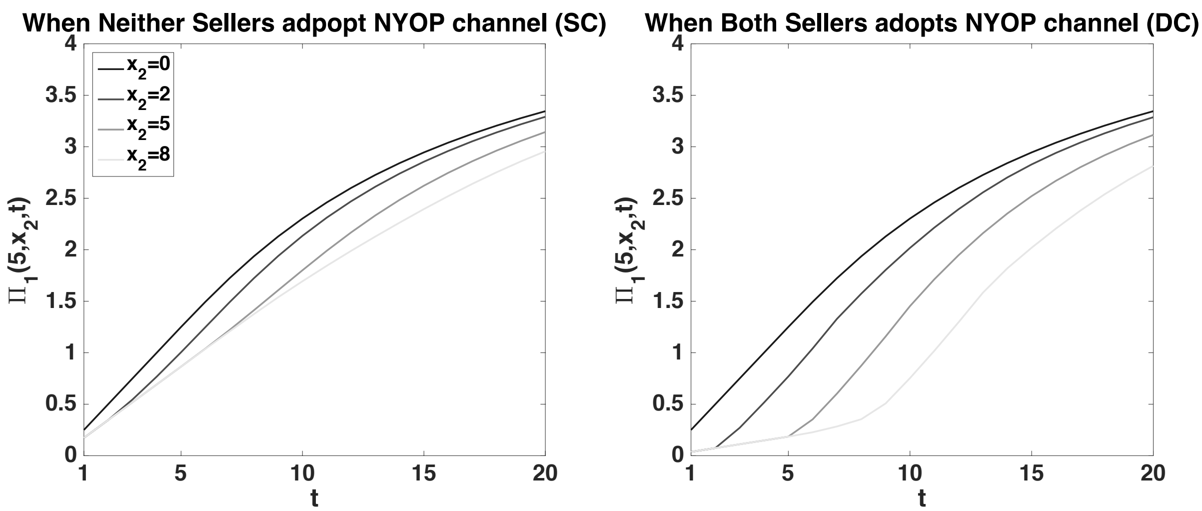

Figure 2 **Expected profit of seller 1 (Π_1) as leftover time (t) varies**

Impact of Inventory Level. In order to illustrate the effect of inventory, we present a set of graphs similar to that in Figure 3. The curves are drawn as a function of one's inventory level (x_1) at time epoch $t = 10$; competitor's inventory level (x_2) is taken from the same range of values ($\{0, 2, 5, 8\}$). For SC, a firm's expected profit increases with his own inventory level, and decreases with competitor's inventory level. Although the latter also holds for DC, the impact of one's own inventory level is somewhat more complicated when both sellers adopt NYOP; we illustrate this through an example illustrated in Figure 3. On the right panel, expected profit for seller 1 (Π_1) retains the increasing-concave shape when competitor's inventory level is low ($x_2 = 0$ or 2); however, as competitor's inventory takes the value $x_2 = 5$, expected profit of seller 1 is actually decreasing when $x_1 \geq 5$, and when $x_2 = 8$, the expected profit Π_1 starts decreasing at $x_1 \geq 2$. The counterintuitive effect in which a firm's expected profit can decline with its own inventory occurs when there is an oversupply ($x_1 + x_2 \geq t$). In a competitive environment with opaque channel, a firm could dispose of extra inventory via NYOP and lower its posted price in order to obtain a larger market share, and it can do it in a credible way that will make it competitor consider this strategy as a credible option. This strategy intensifies the competition between the sellers in both channels, and results in a lower expected profit when the niche opaque channel does not exist.

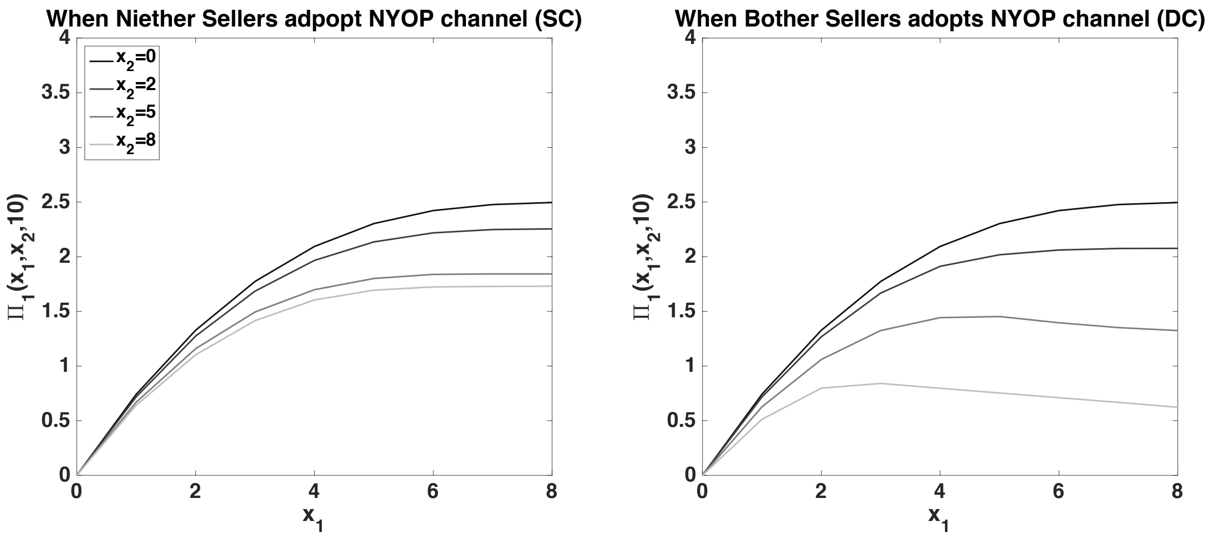

Figure 3 **Expected profit of seller 1 (Π_1) as leftover inventory (x_1) varies**

Impact of Subjective Prior, α_i. Although the existence of equilibrium in DC (Theorem 3) was based on generic prior, $\alpha_1 = \alpha_2 = 0.5$, numerical studies suggest that the result holds for general $\alpha_i \in (0,1)$. To illustrate the impact of subjective priors in DC, we consider three sets of priors (α_1, α_2): $(0.1, 0.9)$, $(0.3, 0.7)$ and $(0.5, 0.5)$, as shown in Figure 4. The top two graphs depict how the profit of seller 1 (Π_1) varies with leftover time (t); the bottom graphs illustrate how seller 1's profit varies with his own inventory level (x_1). In all cases, curves exhibit similar patterns among different priors, implying that non-generic priors do not bring structural changes to our results. On the micro level, numerical findings suggest that the profit of seller 1, in general, decreases with α_1. In other words, seller 1 is better off if customers assume he is less likely to be the opaque product provider than he actually is. Therefore, even if a seller adopts NYOP, it is best for him to "tone down" and not market the fact that his product may be available through the opaque channel as well. This, to a great extent, matches opaque channel's distinction in shielding identity/brand of the sellers.

Impact of Channel Structure. Comparing the left and right panels of Figure 2 and 3, it is apparent that expected profits are generally higher under SC than DC, particularly when the competition is intense (x_1 or x_2 is high, or t is small). Even at $t = 20$, DC generally yields lower expected profit than SC, as summarized in Table 2. The fact that NYOP as an additional channel may actually

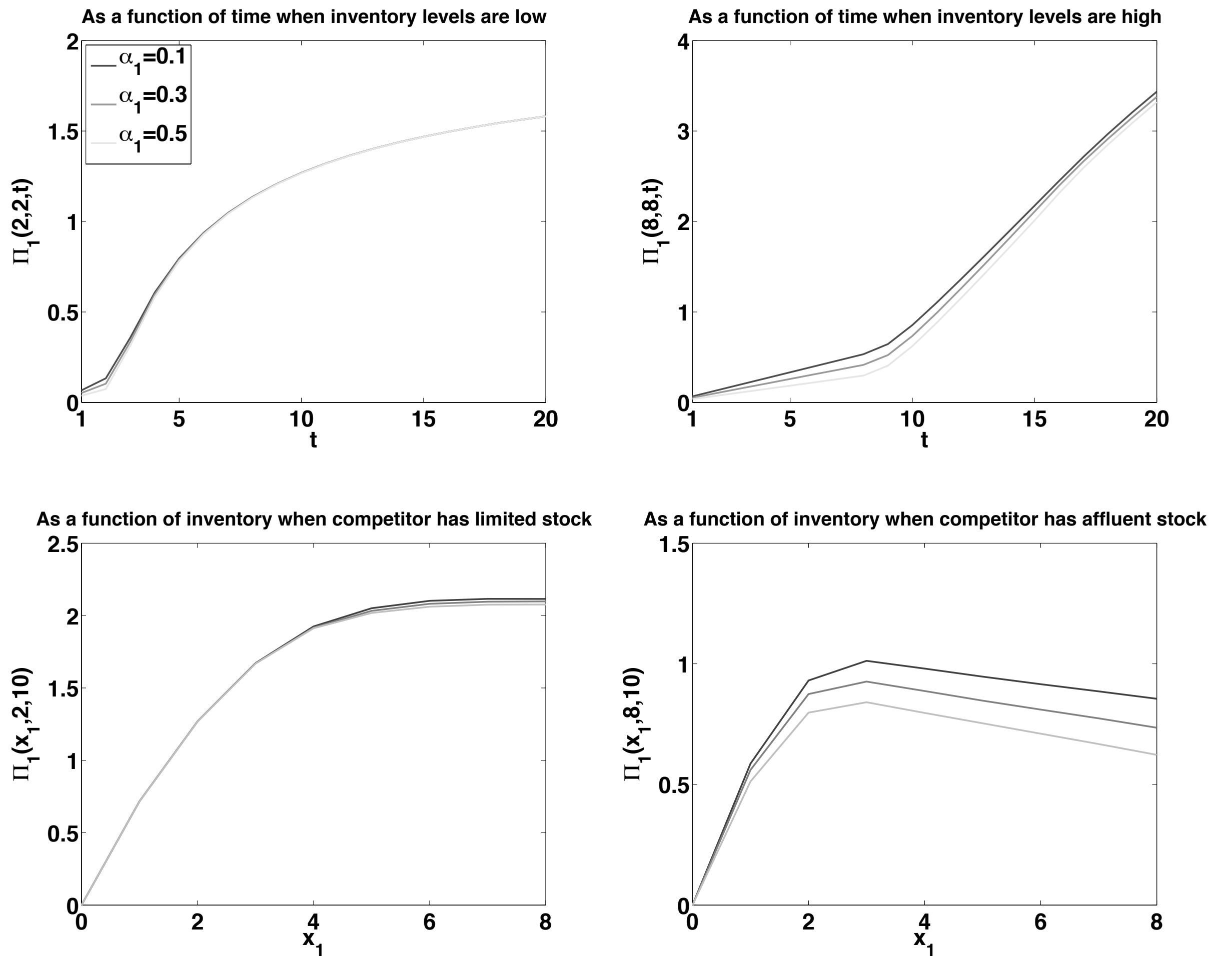

Figure 4 **Expected Profit of Seller 1 (Π_1) as Customer Prior (α_1) varies**

reduce expected profit can be attributed to two reasons. First, the NYOP channel lowers sales revenue by possibly accepting unusually low bids without damaging the integrity of the direct channels. As competition becomes more intense, the reservation price in the NYOP channel could become rather low (eventually becoming $r=0$ in case of over-supply). Thus, there is a reasonable chance that some units will be sold at lower prices in DC than SC. Second, the NYOP channel further intensifies the posted-price competition in the direct channels, which will be discuss further in §6.2.

6.2. Equilibrium Posted Price

Impact of Time and Inventory. In Figures 5 and 6 we plot how a firm's equilibrium posted price varies with time and inventory, using the same set of parameters as in Figure 2 and 3, respectively. As expected, longer time horizon or lower inventory allows higher tag price, and this effect is more evident under DC. For example, at $(x_1, x_2) = (5, 2)$ and t increasing from 1 to 20, the posted price p_1 rises from 0.42 to 0.73 under SC; yet, for DC, p_1 goes up from 0.17 to 0.73. Similarly, at $x_2 = 2$

$100 \times \dfrac{\Pi_1^{SC} - \Pi_1^{DC}}{\Pi_1^{DC}}$		x_2: 0	1	2	3	4	5	6	7	8
x_1	1	0	0.01	0.03	0.06	0.12	0.25	0.54	1.25	2.47
	2	0	0.01	0.04	0.09	0.17	0.35	0.71	1.50	2.98
	3	0	0.02	0.06	0.13	0.25	0.48	0.92	1.81	3.53
	4	0	0.04	0.10	0.21	0.38	0.67	1.20	2.24	4.21
	5	0	0.08	0.21	0.38	0.62	0.99	1.64	2.86	5.13
	6	0	0.21	0.47	0.76	1.12	1.62	2.41	3.81	6.42
	7	0	0.62	1.15	1.68	2.26	2.97	3.94	5.46	8.25
	8	0	1.26	2.34	3.35	4.38	5.49	6.75	8.29	10.37

Table 2 Profit loss (%) for seller 1 under DC vs SC at $t = 20$

and $t = 10$, p_1 drops from 0.84 to 0.5 under SC as x_1 increases from 1 to 8, but from 0.84 only to 0.42 under DC. Overall, it can be observed that equilibrium prices in DC are more sensitive to time horizon and inventory level changes than in SC.

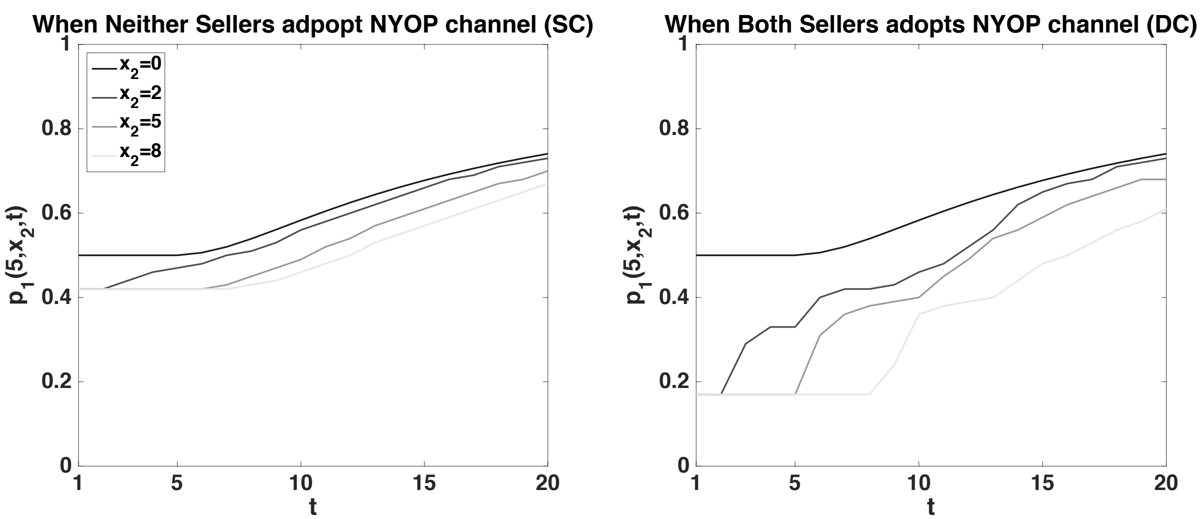

Figure 5 Equilibrium price of seller 1 (p_1) as leftover time (t) varies

Impact of Channel Structure. The two figures above also allow the comparison of equilibrium posted prices between SC (left panel) and DC (right panel). Apparently, prices are lower under DC than SC. To highlight this point, we take a snapshot of the equilibrium prices at $t = 20$ and calculate their differences in Table 3. Our results confirm the intense price competition under DC. Indeed, in the absence of the NYOP channel, sellers compete publicly through posted prices only, and there is less uncertainty about whose product the customer will eventually buy. When the NYOP channel is adopted by both sellers, each of them is aware that his customers can now be cannibalized by his

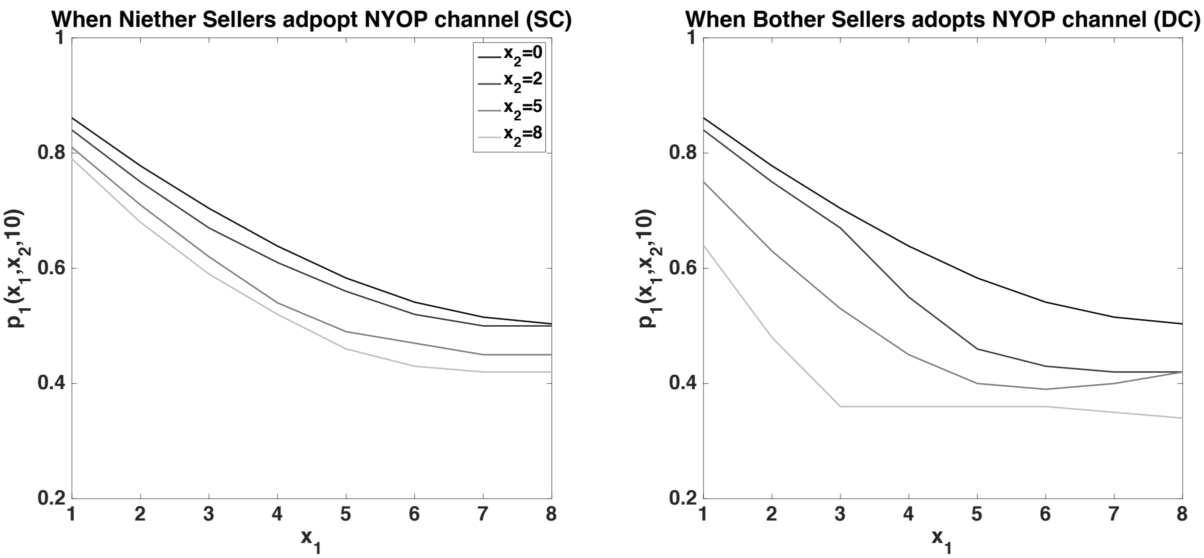

Figure 6 **Equilibrium price of seller 1 (p_1) as leftover inventory (x_1) varies**

		x_2								
$100 \times \dfrac{p_1^{SC} - p_1^{DC}}{p_1^{DC}}$		0	1	2	3	4	5	6	7	8
	1	0	0	0	0	0	0.22	0.33	0.17	2.77
	2	0	0	0	0	0.12	0.24	0.36	2.71	4.55
	3	0	0	0.12	0.12	0.25	0.25	0.64	0.13	6.34
x_1	4	0	0	0.13	0.26	0.40	0.40	0.82	1.39	7.98
	5	0	0.14	0.27	0.42	0.71	1.01	1.32	1.65	10.07
	6	0	0.58	0.88	1.34	1.67	2.17	2.55	2.43	12.91
	7	0	4.94	7.07	7.53	7.67	5.63	5.77	5.36	16.77
	8	0	8.92	13.65	18.09	22.38	26.13	28.96	28.11	15.14

Table 3 **Equilibrium price difference (%) for seller 1 under DC vs SC at $t = 20$**

rival via NYOP, or even by his own participation in the opaque product. It is then not surprising that posted price falls in equilibrium.

Impact of Subjective Prior, α_i. Figure 7 demonstrates how equilibrium price changes when the prior (α_1, α_2) varies among $(0.1, 0.9)$, $(0.3, 0.7)$ and $(0.5, 0.5)$. Similarly to the expected profit, non-generic prior does not change the structure of our results; however, the equilibrium price may be affected in a negative manner. That is, the more customers believes that a seller will be the opaque product provider, the lower is the posted price that this seller can set in equilibrium. In addition, the equilibrium price is more sensitive to the prior when leftover time (t) is short, or when its own inventory level (x_1) is high.

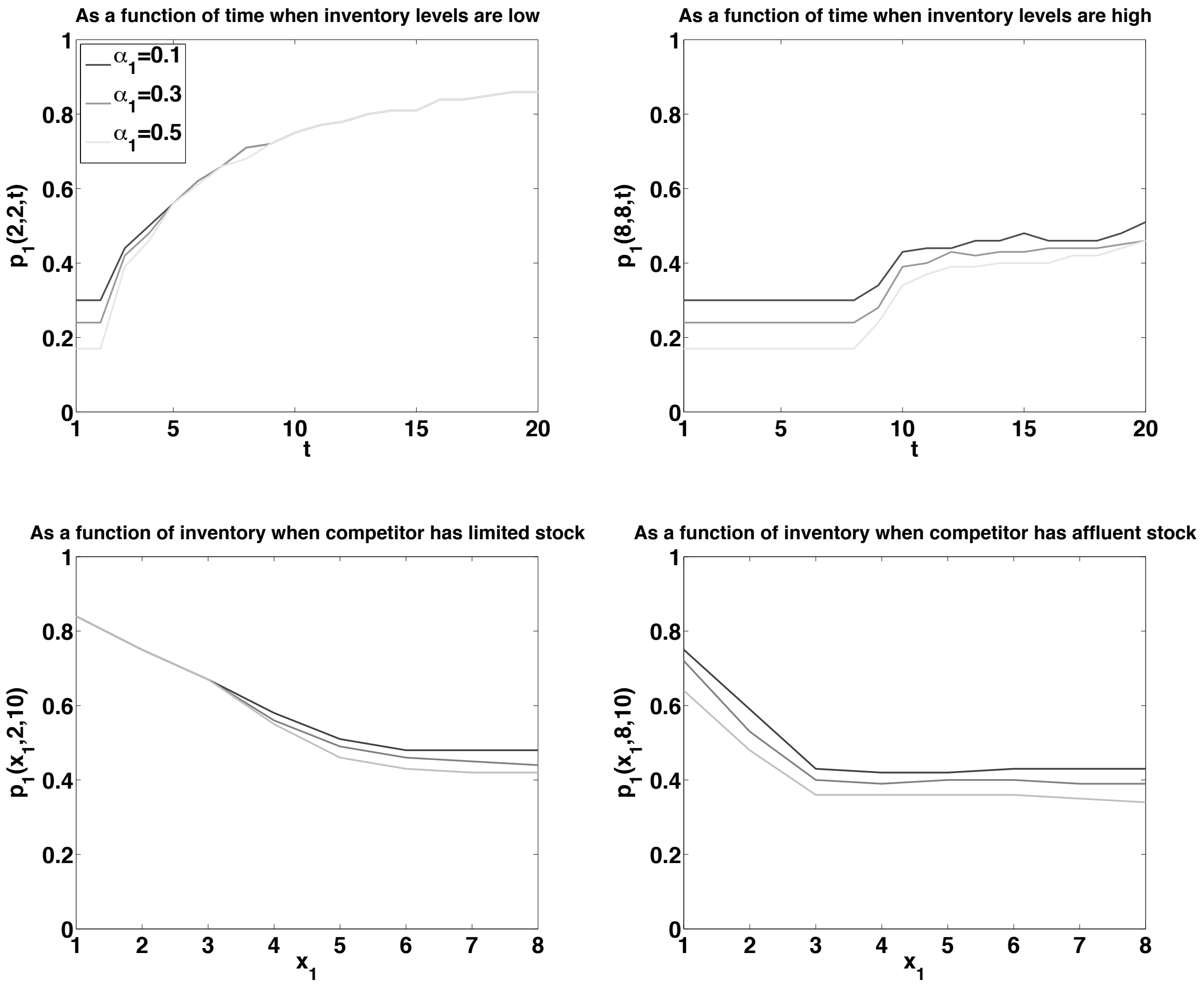

Figure 7 **Equilibrium Price of Seller 1 (p_1) as Customer Prior (α_1) varies**

6.3. Reservation Price

Impact of Time and Inventory. Figure 8 illustrates the reservation price r under DC, where we allow the leftover time, inventory levels, and subjective prior to vary across the four graphs. The time and inventory impacts for the reservation price are similar to that for the posted price—longer leftover time accommodates higher reservation price, and higher inventory level suggests lower reservation price. Therefore, it is more likely for customers to obtain a "deal" in the last minute, or when the market is oversupplied.

Impact of Subjective Prior α_i. The implications for subjective prior, however, are quite different—most often, the reservation price is lower when the belief is biased (α's away from 0.5). That is, if customers believe that one seller is more likely to be the opaque provider, the channel becomes less opaque from the bidder's perspective, which leads to lower reservation price. This suggests that in order to maintain a healthy NYOP environment, it is critical for the intermediary to establish confidence among customers that the offerings are purely "random."

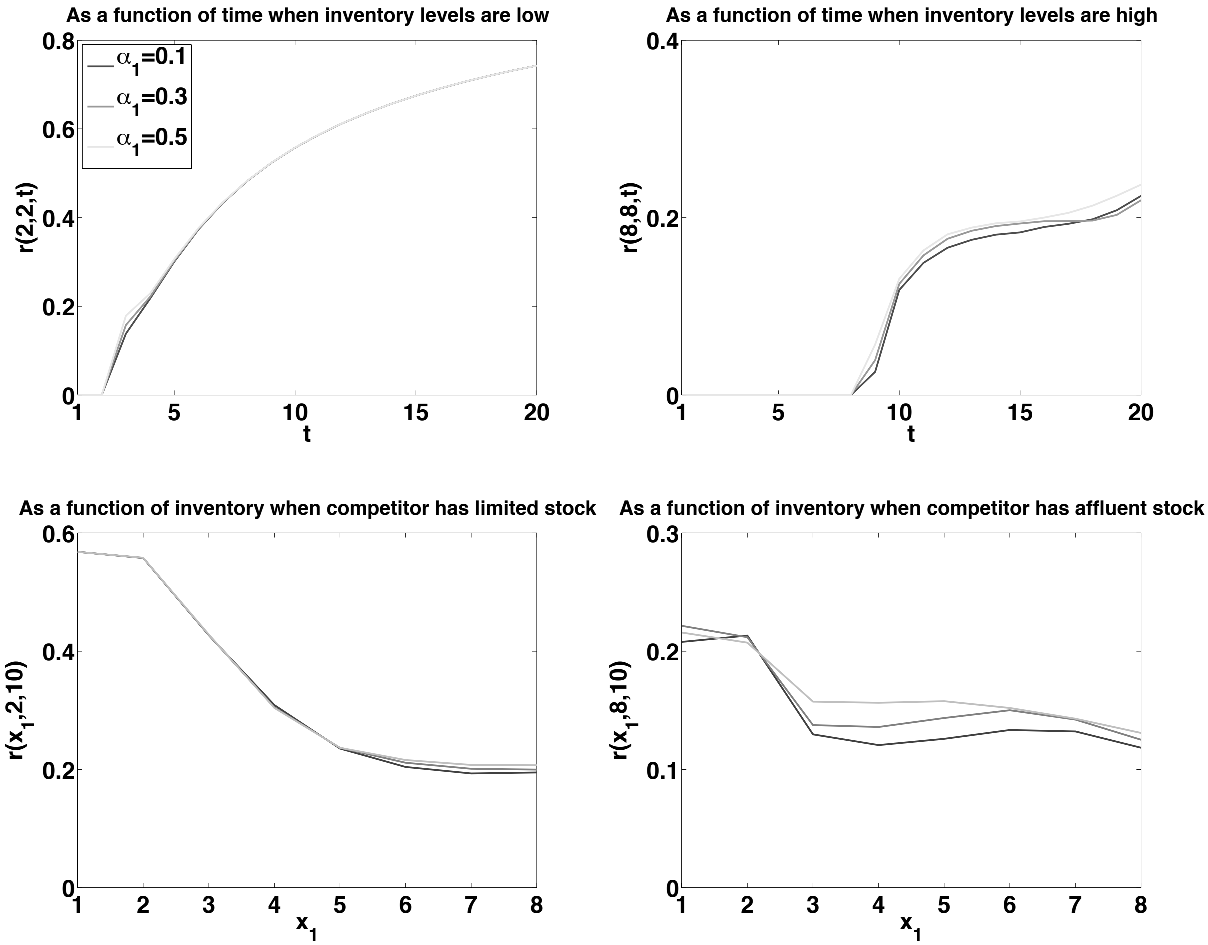

Figure 8 Reservation price as Customer Prior (α_1) varies.

6.4. Equilibrium Channel Decisions

As discussed in §6.1, sellers are generally worse off in presence of NYOP. Thus, a natural question arises: if the NYOP channel is not beneficial, why would the sellers adopt them to begin with? We investigate this question in this subsection.

At the beginning of the season, given the inventory levels $\mathbf{x} = (x_1, x_2)$ and time horizon T, the sellers need to determine their channel strategies. Denote by $S_i = 1$ the case in which seller i decides to adopt NYOP, and $S_i = 0$ if he decide not to do it. Thus $(S_1, S_2) = (0, 0)$ implies the SC structure, $(1, 1)$ the DC structure, $(1, 0)$ the $SDC1$,and $(0, 1)$ the $SDC2$. Seller i's expected profit is denoted by $\Pi_i^{S_1 S_2}(\mathbf{x}, T)$. Thus:

- SC will be the equilibrium channel strategy if $\Pi_1^{00}(\mathbf{x}, T) \geq \Pi_1^{10}(\mathbf{x}, T)$ and $\Pi_2^{00}(\mathbf{x}, T) \geq \Pi_2^{01}(\mathbf{x}, T)$;
- DC will be the equilibrium channel strategy if $\Pi_1^{11}(\mathbf{x}, T) \geq \Pi_1^{01}(\mathbf{x}, T)$ and $\Pi_2^{11}(\mathbf{x}, T) \geq \Pi_2^{10}(\mathbf{x}, T)$;
- $SDC1$, with only seller 1 participating in NYOP, is the equilibrium channel strategy if $\Pi_1^{10}(\mathbf{x}, T) \geq \Pi_1^{00}(\mathbf{x}, T)$ and $\Pi_2^{10}(\mathbf{x}, T) \geq \Pi_2^{11}(\mathbf{x}, T)$;
- and $SDC2$, where only seller 2 participates in NYOP, will be the equilibrium channel strategy

if $\Pi_1^{01}(\mathbf{x},T) \geq \Pi_1^{11}(\mathbf{x},T)$ and $\Pi_2^{01}(\mathbf{x},T) \geq \Pi_2^{00}(\mathbf{x},T)$.

6.4.1. Announced SDCi In the ideal case, where customers are fully aware of each seller's channel strategy, Corollary 1 suggests that $SDCi$ will yield the same outcome as SC; i.e., $\Pi_i^{10}(\mathbf{x},T) = \Pi_i^{01}(\mathbf{x},T) = \Pi_i^{00}(\mathbf{x},T)$. It follows immediately that DC is the equilibrium channel structure if and only if $\Pi_1^{11}(\mathbf{x},T) \geq \Pi_1^{00}(\mathbf{x},T)$ and $\Pi_2^{11}(\mathbf{x},T) \geq \Pi_2^{00}(\mathbf{x},T)$; otherwise, SC is the equilibrium channel structure. Under the same assumptions that we used in §5, we can prove the following result.

Proposition 3. *For unlimited inventory ($x_1 = x_2 = \infty$), SC is the equilibrium when customers are fully informed of sellers' channel strategies.*

Thus, neither seller is motivated to adopt the NYOP channel when there is sufficiently large amount of inventories. This partially explains why NYOP is most often applied in service sectors, wherein supply is rather limited, but seldom observed in production or retailing. Further, numerical results in Table 2 suggest that SC delivers higher expected profit than DC, hence SC would be the equilibrium for any inventory levels. Together with Theorems 1 and 2, this implies that it would be difficult for NYOP to survive in an environment with perfect information and affluent supply.

6.4.2. Generic SDCi Now consider a more realistic case, in which customers cannot observe sellers' channel strategies and would adopt the generic subjective prior in estimating the opaque product structure. Unlike the announced SDCi, the generic SDCi no longer yields the same outcome as SC. In fact, it is possible that a seller utilizes both the direct and opaque channels at the same time. In this case, seller 1 would compare his expected profit in DC with the one under generic SDC2 in order to decide whether to stay in DC. From Table 4, we find that the decision can be mixed — there is no strict preferential between DC or SDC2 for seller 1; the decision depends on inventory levels and leftover time. The same logic applies to other channel structures; therefore, the equilibrium channel structure exhibits more variety under generic SDCi than under announced SDCi.

We pick four time lengths, $t = 18, 12, 5, 2$, to mimic long versus short selling seasons, and depict the equilibrium channel strategy in Figure 9. One can observe that the equilibrium outcome heavily depends upon the initial inventory levels and the length of selling horizon. For example, when the time horizon is short ($t = 2$), sellers may differentiate themselves via channel strategies: specifically, only one of them will adopt dual channels, and the other will sell through direct channel only. The same holds true when the time line is moderately short ($t = 5$), except that single channel strategy may become the NE if supply is scarce for at least one seller. Therefore, even though the marginal value of the inventory is low in the last minute, sellers will still be able to avoid a pricing war by proper channel differentiation. This insight persists as the supply increases. In fact, it can be proved that:

$100 \times \dfrac{\Pi_1^{SDC2} - \Pi_1^{DC}}{\Pi_1^{DC}}$		x_2 = 1	2	3	4	5	6	7	8
	1	-0.04	-0.06	-0.07	-0.03	0.08	0.34	1.02	2.18
	2	-0.06	-0.10	-0.12	-0.09	0.03	0.34	1.09	2.50
	3	-0.07	-0.12	-0.14	-0.11	0.01	0.37	1.18	2.75
x_1	4	-0.04	-0.09	-0.12	-0.08	0.08	0.46	1.38	3.15
	5	-0.01	-0.01	0.03	0.11	0.32	0.77	1.78	3.84
	6	0.12	0.23	0.37	0.56	0.83	1.42	2.65	5.00
	7	0.50	0.87	1.22	1.59	2.13	2.94	4.35	6.93
	8	1.11	2.01	2.83	3.70	4.69	5.87	7.36	9.45

Table 4 Profit loss (%) for seller 1 under DC vs generic SDC2 at $t = 20$

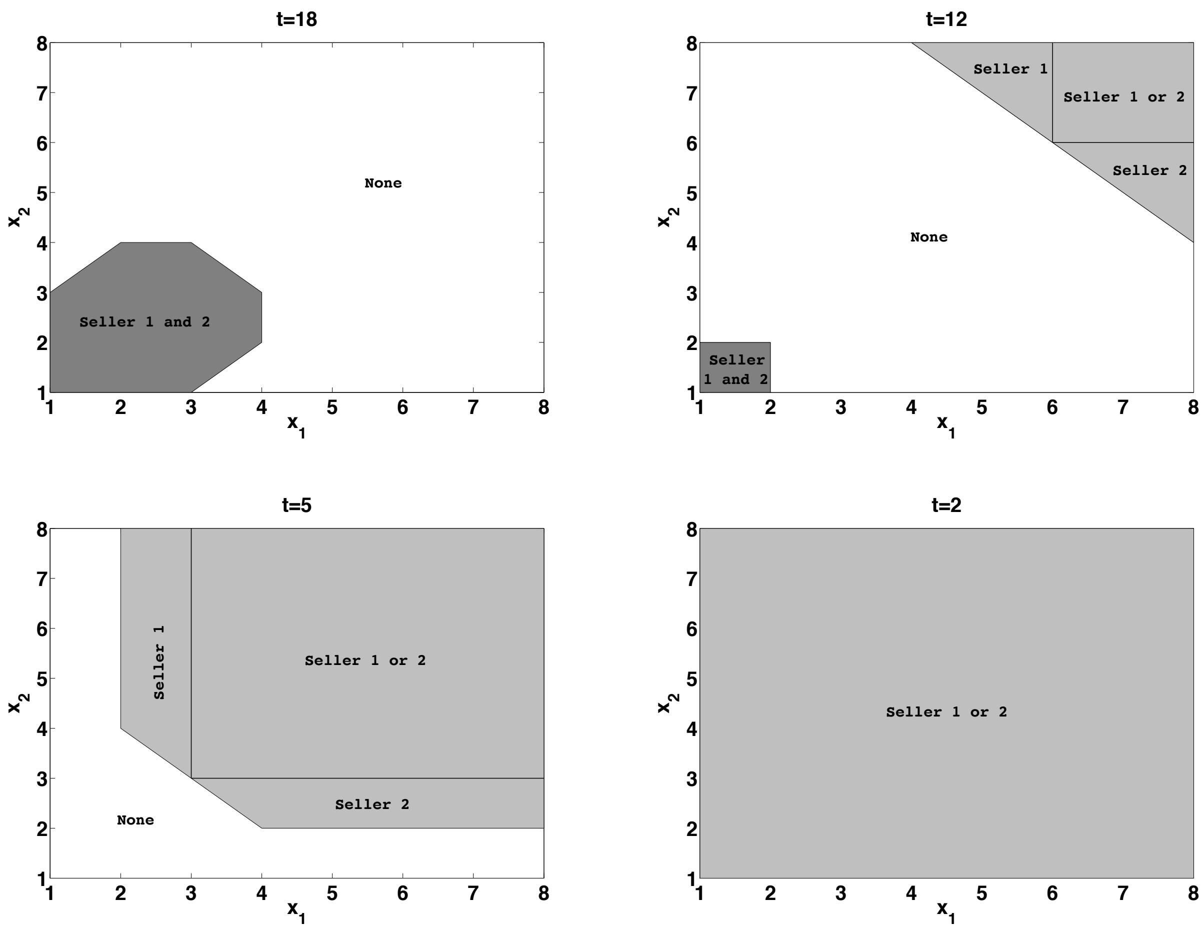

Figure 9 Equilibrium channel strategy: who will sell through the NYOP channel?

PROPOSITION 4. *For unlimited inventory ($x_1 = x_2 = \infty$), SDCi is an equilibrium when sellers' channel strategies are opaque to the customers.*

Thus contrary to Proposition 3, channel opaqueness encourages some seller to adopt NYOP when supply is not a critical issue. Therefore, for industries such as production and retailing to consider the NYOP option, one needs to increase the opaqueness of the offerings, .e.g, restrict customers from choosing the color of an item, create private label brand, etc.

While high inventory level to some extent facilitates the adoption of NYOP, this effect could be offset by the long selling horizon. As the top graphs with $t = 18$ and $t = 12$ suggest, the SDCi channel differentiation vanishes with prolonged selling horizon. Indeed, with moderate inventory level and sufficient future demand, a seller finds himself less pressured to sell his products via a discounted channel if his rival does not.

The area that attracts most of our interest is when DC becomes the equilibrium; i.e., both seller 1 and 2 adopt NYOP channel. We observe that in the darkest area of Figure 9, both DC and SC are equilibria. That is to say, when the selling horizon is long and inventory level at both sellers are low, it is possible for both sellers to adopt NYOP and direct channels at the same time. Since this is not a last minute sale ($t >> 0$), the reservation price can be reasonably high. Hence, a seller may benefit by competing on the opaque platform rather than losing customers to a dual-channel rival.

Throughout the analysis in this subsection, we find that customers' information about sellers' channel strategy can be a deal breaker for the existence of NYOP channel. Namely, when the NYOP channel fully shields sellers' channel strategies, depending on the timing and inventory levels, all channel structures can appear in equilibrium. Sellers can properly use NYOP channel in differentiating themselves during last minutes sale, or protecting their market ground at the beginning of the selling season. In either case, zero reservation price (or free give-away) can be blocked by rational channel choice between the sellers.

7. Discussion and Extensions

Businesses today are under increasing pressure to refine their distribution channel and pricing strategies. In this study, we aim to understand how the use of NYOP opaque channel may impact the industry with direct sales channels. We develop a model framework that accommodates a full line of stakeholders, including the upstream sellers, the NYOP intermediary firm, and the downstream customers. The model allows explicit analysis of the optimal/equilibrium decisions, and provides important managerial implications for each stakeholder. We also illustrate how inventory level, time horizon, and information structure may affect sellers' channel strategies. To the best of our knowledge, this is one of the first papers that consider operations issues for the NYOP-opaque-channel problem in a competitive environment.

While our paper deals with a stylized model, we believe that most of our assumptions can be validated, and there are many directions in which this work can be extended. We discuss a few of them below.

7.1. Opaque Product Design

The underlying products of the opaque offerings are similar in general. For example, they can be same-star-level hotels in a common region, or flights with the same departing/landing dates. However, in terms of mass consumer valuation, these two cases can be quite different. For hotels, it is natural that some customers may prefer brand A to brand B, while others have an opposite taste. There is usually no common preference for this kind of products/services. Yet for flights, prime-time flights are normally considered better than red-eye flights, both of which may appear in the opaque product realization. Thus, customers hold common preferences upon this type of product/service.

We approximate the above two scenarios by horizontal and vertical differentiation between the two products. In case where no common consensus could be reached upon the superiority of a product, we talk about *horizontal* differentiation and assume that $v_{1t} + v_{2t} = \bar{v}_t$ for some constant $\bar{v}_t > 0$ and $v_{1t} \sim U[0, \bar{v}_t]$. The model is then reduced to a simple Hotelling model with surplus $\bar{s} = \bar{v}_t$ and linear transportation cost $t = \bar{v}_t$ (Tirole 1988).

When customers commonly prefer one product to the other, we assume *vertical* differentiation and assume that $v_{1t} - v_{2t} = \bar{v}_t$, for some constant $\bar{v}_t$. It can be shown that these two scenarios have very different implications for the intermediary firm.

PROPOSITION 5. **(Underlying Products: horizontal vs. vertical differentiation)**

(i). If the products are vertically differentiated and $v_{it} > v_{jt}$, then seller i will set a posted price such that all customers purchase from its direct channel; the intermediary NYOP firm will collect zero rents.

(ii). If the products are horizontally differentiated, then there exists $p_t^0 > 0$ such that (a) when the minimum posted price, p_t, satisfies $p_t > p_t^0$, all customers will be covered, and the intermediary NYOP firm earns positive revenue; (b) when $p_t \leq p_t^0$, some customers will be left empty-handed, and no purchase will be finalized through NYOP.

This implies that, if possible, the intermediary firm should carefully select the underlying products for the opaque offering. If the product candidates have a clear quality difference (a flight departing at 1 a.m. vs. a flight on the same route that leaves at noon), or one product has better brand recognition (such that all customers may prefer one particular product, although the star levels are the same), then the intermediary firm may not benefit much from these differences. The main reason is that the seller with “better quality/image” can obtain the valuation difference via proper posted price setting, at which the customers feel that the opaque product is not worth bidding for. On the other hand, if the products are horizontally differentiated (e.g., hotels with the same star ranking but different brand-name or clustered close-by), the valuation for the underlying product will be much more diversified. The sellers would then need the intermediary firm to shoulder part of this uncertainty, so that their direct channels can focus on their own target customers. Overall, the intermediary benefits more from horizontally differentiated products than from vertically differentiated ones. This insight conforms with Priceline's 2011 Annual Report to Stockholders (pp.55) stating that NYOP sales increased in hotel rooms and rental cars, while they declined in air tickets.

As the number of sellers increases, there is more leverage for the intermediary firm to determine its supplier base and refine its opaque product design. The firm may also choose whether to inform the customers about the underlying products that comprise the opaque product, or blind it (Bai et al. 2015). There are many interesting research/practical questions with regard to this line of decision making.

7.2. Contracting

Throughout the paper, we consider the intermediary firm as a revenue maximizer, which always pairs a bid with the seller that charges a lower reservation price. As a result, §6.2 shows that the sellers' channel strategy can take many forms, including that neither will sell through the NYOP intermediary. Therefore, it is worthwhile for the intermediary firm to reconsider the terms and mechanisms in selecting the opaque product provider. In essence, the intermediary should weight the short-run profit earning versus the long-run participation rate of the sellers. These may involve revenue sharing contract between the NYOP firm and the sellers, selecting opaque provider on a randomized basis rather than a reservation-price basis, etc.

7.3. Information Availability

A key distinction between opaque product and regular product is item-information revelation. §6.4 highlights the importance of information on the adoption of NYOP. It is, then, in the interest of the intermediary firm to explore to what degree should information be offered to its customers. For example, customers purchasing from Hotwire.com can see other customers' reviews, amenities, and potential brand names of the blinded hotel. Yet at Priceline.com, fairly little additional information is available for the customers during their decision making. However, for close yet unsuccessful bids, Priceline may give a one-time "counter-offer," hinting the upper bound of the reservation price. New York City-startup Stayful also provides customers with a likely-to-accept rate as bidding reference (Tnooz.com 2013). Given that the two different pricing mechanism (seller-driven vs. buyer-driven), it is interesting to consider the level and timing of information-revelation under both settings.

We have also assumed that customers cannot observe real-time inventory levels at the sellers. One may consider whether it is worthwhile for the sellers to signal their own inventory levels, as observed in recent practice of some companies i aimed to create a sense of scarcity, and possibly at reducing strategic behavior.

7.4. Consumer Behavior

Modelling the way consumers comprehend the NYOP/opaque channel can be quite challenging due to the complexity of the system. At the time of this paper, the approaches are still open and depending on the context, customers have been allowed to be generic (Fay 2008), bounded-rational via learning (Huang and Yu 2014) or fully rational (Chen et al. 2014). It is to future research's interest to examine to what extent these assumptions/models hold true to actual consumer behavior. In this aspect, Huang et al. (2014) designed an experiment supporting the generic assumptions used in §6. The experimental results also suggest that customers tend to overbid when their valuations are high, and the bids fluctuate most when only partial information is provided. These behavioral phenomena deserve further analytical exploration.

7.5. Social Media

Another promising direction is to consider consumer behavior under the influence of social media. Today's Internet-based applications allow customers to review, discuss, and exchange information beyond traditional online shopping. Specifically for the NYOP channel itself, customers have been sharing successful bidding outcomes on several websites (betterbidding.com, hoteldealsrevealed.com, biddingfortravel.yuku.com, facebook.com, etc.). This can trigger another line of study regarding strategic customer behavior, which includes social learning, bidding postponement, repeated bidding, etc. Whether the NYOP intermediary should publicize all (or part of) the recent bids, or analyze the historical bidding data and take any counteraction, remain unexplored research questions.

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Appendix

Proof of Proposition 1

For a customer who can afford neither product at listed price ($v_i < p_i$ for $i = 1,2$), NYOP is the only channel in which she might get some product, so she will always attend NYOP. Her expected payoff is $V_{DC}^B(\mathbf{v}) = \max_b G(b)(\alpha_i v_j + \alpha_i v_j - b)$ The FOC of the inner function is $g(b)(\alpha_1 v_1 + \alpha_2 v_2 - b) - G(b)$. Under DRHR assumption, there exists a unique $b^* \in [0, p]$ that satisfies

$$\alpha_1 v_1 + \alpha_2 v_2 = b^* + G(b^*)/g(b^*). \tag{A1}$$

Also, as $G(b)/g(b)$ increases in b, the optimal bid increases with the expected valuation of the opaque product $\alpha_1 v_1 + \alpha_2 v_2$.

For a customer with an external choice ($v_1 \geq p_1$ and/or $v_2 \geq p_2$), the optimal bid b^* maximizes $G(b)(\alpha_1 v_1 + \alpha_2 v_2 - b) + \bar{G}(b) \max_{k=0,1,2,} V_{DC}^k(\mathbf{v})$. Without loss of generality, assume that $v_1 - p_1 \geq v_2 - p_2$, and denote $|v_i - v_j + p_j - p_i| = v_1 - v_2 + p_2 - p_1 > 0$ the *degree of differentiation*. Followed by a similar analysis as above, it is the unique value that satisfies

$$\alpha_2(v_2 - v_1) + p_1 = b^* + G(b^*)/g(b^*). \tag{A2}$$

Due to DRHR assumption, $b^* \geq 0$ if and only if $\alpha_2(v_2 - v_1) + p_1 \geq 0$. The bid increases with $v_2 - v_1$, hence decreasing in $v_1 - v_2 + p_2 - p_1$, the degree of differentiation.

The customer will choose to bid b^* at the NYOP channel first, if and only if it yields higher expected payoff than buying from direct channel 1, i.e., $V_{DC}^B(\mathbf{v}) \geq V_{DC}^1(\mathbf{v})$, which requires that $G(b^*)(\alpha_1 v_1 + \alpha_2 v_2 - b^*) + \bar{G}(b^*)(v_1 - p_1) > v_1 - p_1$, or equivalently,

$$b^* < (\alpha_1 v_1 + \alpha_2 v_2) - (v_1 - p_1) = \alpha_2(v_2 - v_1) + p_1. \tag{A3}$$

This apparently holds when $\alpha_2(v_2 - v_1) + p_1 \geq 0$. Therefore, the customer will NYOP first if and only if $\alpha_2(v_2 - v_1) + p_1 > 0$, which is equivalent to requiring that the degree of differentiation $v_1 - v_2 + p_2 - p_1 \leq p_1/\alpha_2 + p_2 - p_1 = \dfrac{\alpha_1 p_1 + \alpha_2 p_2}{\alpha_2}$. □

Proof of Theorem 1

Suppose only seller 1 is in stock, the product at the intermediary firm ceases to be an opaque product and becomes a regular one. It is obviously beneficial for the customers to NYOP in the first place, and if rejected, reconsider purchasing from the direct channel. Therefore, the customer first has to determine her bid, b, that maximizes her expected payoff:

$$G(b)(v_j - b) + \bar{G}(b) \max\{v_1 - p_1, 0\} = \begin{cases} G(b)(p - b) + (v_1 - p_1), & \text{if } v_1 \geq p_1 \\ G(b)(v_1 - b), & \text{if } v_1 < p_1. \end{cases}$$

The FOC is given by

$$g(b)\left[\min\{v_1,p_1\}-b\right]-G(b)=g(b)\left[\min\{v_1,p_1\}-b-\frac{G(b)}{g(b)}\right].$$

As $G(b)/g(b)$ increases in b, there exists a unique $b^*\in[0,\min\{v_1,p_1\}]$ such that $\min\{v_1,p_1\}-b^*-\frac{G(b^*)}{g(b^*)}=0$. It is easy to verify that the extreme solutions ($b=0$ and $b=\min\{v,p\}$) are not optimal; therefore, the expected payoff is maximized at FOC=0, i.e, when $b=b^*$. This raises the following lemma:

Lemma A1. *When only seller 1 is in stock with direct channel price p_1, a customer with valuation v_1 will place a bid b^* with the NYOP channel in the first place, which satisfies*

$$b^*+\frac{G(b^*)}{g(b^*)}=\min\{v_1,p_1\} \tag{A4}$$

By (A4), the optimal bid satisfies $b^*+\frac{G(b^*)}{g(b^*)}=\min\{v_1,p_1\}\le p_1$. Consider r_0 which satisfies

$$r_0+\frac{G(r_0)}{g(r_0)}=p_1. \tag{A5}$$

Then, obviously $b^*<r_0$, i.e., r_0 is beyond the bid of any customer. Therefore if $r\ge r_0$, a customer will be always rejected by the NYOP channel and if $v_1\ge p_1$, she will obtain the product through the direct channel. On the other hand, if $r<r_0$, a customers with $v_1\ge r+\frac{G(r)}{g(r)}$ will bid over r and be accepted by the NYOP intermediary. If one is rejected for bidding below r, i.e., $v_1<r+\frac{G(r)}{g(r)}$, she cannot afford the posted price neither since $v_1<r+\frac{G(r)}{g(r)}<r_0+\frac{G(r_0)}{g(r_0)}=p$. This proves the following lemma:

Lemma A2. *At period t, suppose r_0 is defined by* (A5) *with $p_1=p_{1t}$. The sales will only be realized through the direct channel if $r_t\ge r_0$, or the NYOP channel otherwise.*

Now consider the seller's optimal decision. With posted price p and reservation price r for a particular period, seller 1's spot revenue π_1 is given by

$$\pi_1(p,r)=\begin{cases}\bar F_1(p)p, & \text{if } r>r_0\\ \bar F_1\left(r+\frac{G(r)}{g(r)}\right)r & \text{if } r\le r_0,\end{cases}$$

where r_0 is defined in (A5).

At $t=1$, seller 1 will seek a pair (p,r) that maximizes $\pi_1(p,r)$. Denote p^* the optimal posted price if the seller determines to use direct channel, i.e., $r\ge r_0$. If the seller chooses to use the NYOP channel ($r<r_0$) only, the objective becomes

$$\max_{r<r_0}\bar F_1\left(r+\frac{G(r)}{g(r)}\right)r=\max_{r<r_0}\frac{r}{r+G(r)/g(r)}\bar F_1\left(r+\frac{G(r)}{g(r)}\right)\left(r+\frac{G(r)}{g(r)}\right)$$

$$\leq \max_{r<r_0} \frac{r}{r+G(r)/g(r)} F_1(p^*)p^*$$
$$< F_1(p^*)p^*.$$

Therefore, in the last time epoch it is always optimal not to let customers purchase from the NYOP channel.

At $t>1$, denote by $\Pi_1(x,t)$ the optimal expected profit that seller 1 will receive if inventory level is x. Then,

$$\Pi_1(x,t) = \max_{p\geq r\geq 0} \begin{cases} \bar{F}_1(p)\,[\,p+\Pi_1(x-1,t-1)\,]+F_1(p)\Pi_1(x,t-1), & \text{if } r>r_0 \\ \bar{F}_1\left(r+\dfrac{G(r)}{g(r)}\right)[\,r+\Pi_1(x-1,t-1)\,]+F_1\left(r+\dfrac{G(r)}{g(r)}\right)\Pi_1(x,t-1) & \text{if } r\leq r_0, \end{cases}$$

Let r^* be the optimal reservation price if the seller decides to go with NYOP selling, i.e.,

$$r^* = \arg\max_r \bar{F}_1\left(r+\frac{G(r)}{g(r)}\right)[\,r+\Pi_1(x-1,t-1)\,]+F_1\left(r+\frac{G(r)}{g(r)}\right)\Pi_1(x,t-1).$$

Consider $p^* = r^* + \dfrac{G(r^*)}{g(r^*)} - \epsilon < r^* + \dfrac{G(r^*)}{g(r^*)}$, where $\epsilon \to 0$. The following verifies that selling through direct channel with posted price $p_t = p^*$ yields a higher expected profit then that can be achieved via NYOP selling only:

$$\begin{aligned}
&\bar{F}_1(p^*)\,[\,p^*+\Pi_1(x-1,t-1)\,]+F_1(p^*)\Pi_1(x,t-1) \\
&= \bar{F}_1\left(r^*+\frac{G(r^*)}{g(r^*)}\right)[r^*+\Pi_1(x-1,t-1)]+F_1\left(r^*+\frac{G(r^*)}{g(r^*)}\right)\Pi_1(x,t-1)+\bar{F}_1\left(r^*+\frac{G(r^*)}{g(r^*)}\right)\frac{G(r^*)}{g(r^*)} \\
&+\left[\bar{F}_1\left(r^*+\frac{G(r^*)}{g(r^*)}-\epsilon\right)-\bar{F}_1\left(r^*+\frac{G(r^*)}{g(r^*)}\right)\right]\left[r^*+\frac{G(r^*)}{g(r^*)}+\Pi_1(x-1,t-1)-\Pi_1(x,t-1)\right] \\
&-\epsilon\bar{F}_1\left(r^*+\frac{G(r^*)}{g(r^*)}-\epsilon\right).
\end{aligned}$$

When ϵ is small enough, we have

$$\begin{aligned}
&\left[\bar{F}_1\left(r^*+\frac{G(r^*)}{g(r^*)}-\epsilon\right)-\bar{F}_1\left(r^*+\frac{G(r^*)}{g(r^*)}\right)\right]\left[r^*+\frac{G(r^*)}{g(r^*)}+\Pi_1(x-1,t-1)-\Pi_1(x,t-1)\right] \\
&-\epsilon\bar{F}_1\left(r^*+\frac{G(r^*)}{g(r^*)}-\epsilon\right)\to 0.
\end{aligned}$$

Hence,

$$\begin{aligned}
&\bar{F}_1(p^*)\,[\,p^*+\Pi_1(x-1,t-1)\,]+F_1(p^*)\Pi_1(x,t-1) \\
&\geq \bar{F}_1\left(r^*+\frac{G(r^*)}{g(r^*)}\right)[r^*+\Pi_1(x-1,t-1)]+F_1\left(r^*+\frac{G(r^*)}{g(r^*)}\right)\Pi_1(x,t-1).
\end{aligned}$$

Therefore, it is optimal for seller 1 to use direct channel only.

We characterize the optimal price and expected profit as follows:

For $t = 1$, $p_{11}^* = \arg\max_p \bar{F}_1(p)p = \arg\max_p (1-p)p = 1/2$, $\Pi_1(x,1) = \bar{F}_1(p_{11}^*)p_{11}^* = 1/4$.

For $t > 1$,

$$\begin{aligned} p_{1t}^*(x) &= \arg\max_p \bar{F}_1(p)\,[p + \Pi_1(x-1,t-1)] + F_1(p)\Pi_1(x,t-1) \\ &= \arg\max_p [1-p]\,[p + \Pi_1(x-1,t-1)] + p\Pi_1(x,t-1) \\ &= \arg\max_p -p^2 + p[\,1 + \Pi_1(x,t-1) - \Pi(x-1,t-1)\,] + \Pi_1(x-1,t-1) \\ &= \frac{1 + \Pi_1(x,t-1) - \Pi_1(x-1,t-1)}{2} \end{aligned}$$

and

$$\begin{aligned} \Pi_1^*(x,t) &= \bar{F}_1(p_{1t}^*)\,[p_{1t}^* + \Pi_1(x-1,t-1)] + F_1(p_{1t}^*)\Pi_1(x,t-1) \\ &= \Pi_1(x-1,t-1) + \left(\frac{1 + \Pi_1(x,t-1) - \Pi_1(x-1,t-1)}{2}\right)^2 \end{aligned}$$

□

Proof of Theorem 2:

(i) Under the uniform valuation assumption, the final purchasing realization (1) is

$$\mathbf{H}_{SDC1}(\mathbf{v}) = \begin{cases} (0,0) & \text{if } \max\{v_1 - p_1, v_2 - p_2\} < 0 \text{ and } 2r > v_1; \\ (1,p_1) & \text{if } v_1 - p_1 \geq \max\{v_2 - p_2, 0\} \text{ and } p_1 - 2r < 0; \\ (2,p_2) & \text{if } v_2 - p_2 \geq \max\{v_1 - p_1, 0\} \text{ and } p_2 - 2r < v_2 - v_1; \\ (O,b^*) & \text{otherwise.} \end{cases} \tag{A6}$$

Specifically, if $2r \geq p_1$,

$$\mathbf{H}_{SDC1}(\mathbf{v}) = \begin{cases} (0,0) & \text{if } \max\{v_1 - p_1, v_2 - p_2\} < 0; \\ (1,p_1) & \text{if } v_1 - p_1 \geq \max\{v_2 - p_2, 0\}; \\ (2,p_2) & \text{if } v_2 - p_2 \geq \max\{v_1 - p_1, 0\} \end{cases} \tag{A7}$$

otherwise if $2r < p_1$,

$$\mathbf{H}_{SDC1}(\mathbf{v}) = \begin{cases} (0,0) & \text{if } \max\{v_1 - p_1, v_2 - p_2\} < 0 \text{ and } 2r > v_1; \\ (2,p_2) & \text{if } v_2 - p_2 \geq \max\{v_1 - p_1, 0\} \text{ and } p_2 - 2r < v_2 - v_1; \\ (O,b^*) & \text{otherwise.} \end{cases} \tag{A8}$$

It can be verified that for every $(p_1, r_1) = (1, r)$, the expected profit Π_1 could be enhanced by setting $(p_1, r_1) = (2r, 1)$. Therefore it is optimal for seller 1 to sell through direct channel only.

(ii) We only need to prove the existence of equilibrium on (p_{1t}, p_{2t}) when seller 1 choose not to sell through NYOP (i.e., $r_{1t}^* = 1$, $\Pi_{1t}^* = \Pi_{1t}^D$) in period t. In particular, when $r_{1t} = r_{2t} = 1$,

$$\Pi_1^D(\mathbf{x},t) = [\,p_{1t} + \Pi_1(x_1 - 1, x_2, t-1)\,]\,\Omega_1(\mathbf{r}_t, \mathbf{p}_t) + \Pi_1(x_1, x_2 - 1, t-1)\Omega_2(\mathbf{r}_t, \mathbf{p}_t) + \Pi_1(x_1, x_2, t-1)\Omega_0(\mathbf{r}_t, \mathbf{p}_t)$$

$$\Pi_2(\mathbf{x},t) = \Pi_2(x_1-1,x_2,t-1)\Omega_1(\mathbf{r}_t,\mathbf{p}_t)+[\,p_{2t}+\Pi_2(x_1,x_2-1,t-1)\,]\,\Omega_2(\mathbf{r}_t,\mathbf{p}_t)+\Pi_2(x_1,x_2,t-1)\Omega_0(\mathbf{r}_t,\mathbf{p}_t)$$

We need to prove that Π_1^D and Π_2 are unimodals in p_{1t} and p_{2t} respectively. Note that for $i=1,2$,

$$\frac{\partial\Omega_1(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_i}+\frac{\partial\Omega_2(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_i}+\frac{\partial\Omega_0(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_i}=0.$$

Thus,

$$\begin{aligned}\frac{\partial\Pi_1^D(\mathbf{x},t)}{\partial p_{1t}} &= \Omega_1(\mathbf{r}_t,\mathbf{p}_t)\\ &\quad -\left[\widetilde{\Pi}_1(x_1,x_2,t-1)-p_{1t}\right]\frac{\partial\Omega_1(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{1t}}-[\Pi_1(x_1,x_2-1,t-1)-\Pi_1(x_1,x_2,t-1)]\frac{\partial\Omega_0(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{1t}}\end{aligned}$$

$$\begin{aligned}\frac{\partial\Pi_2(\mathbf{x},t)}{\partial p_{2t}} &= \Omega_2(\mathbf{r}_t,\mathbf{p}_t)\\ &\quad -\left[\widetilde{\Pi}_2(x_1,x_2,t-1)-p_{2t}\right]\frac{\partial\Omega_2(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{2t}}-[\Pi_2(x_1-1,x_2,t-1)-\Pi_2(x_1,x_2,t-1)]\frac{\partial\Omega_0(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{2t}}\end{aligned}$$

By (A6) for $r=1$, it can be verified that if $p_i\le p_j$,

$$\Omega_i(\mathbf{r},\mathbf{p})=1-\frac{2p_i+(1-p_j)^2}{2},\ \ \Omega_j(\mathbf{r},\mathbf{p})=\frac{(2p_i+1-p_j)(1-p_j)}{2},\ \ \Omega_0(\mathbf{r},\mathbf{p})=p_1p_2,\ \ \Omega_O(\mathbf{r},\mathbf{p})=0$$

and

$$\begin{aligned}&\frac{\partial\Omega_i(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{it}}=-1,\quad \frac{\partial\Omega_j(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{it}}=1-p_j,\quad \frac{\partial\Omega_0(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{it}}=p_j;\\ &\frac{\partial\Omega_i(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{jt}}=1-p_j,\quad \frac{\partial\Omega_j(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{jt}}=p_j-1,\quad \frac{\partial\Omega_0(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{jt}}=p_i;\\ &\frac{\partial^2\Omega_i(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{it}^2}=0,\quad \frac{\partial^2\Omega_j(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{it}^2}=0,\quad \frac{\partial^2\Omega_0(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{it}^2}=0;\\ &\frac{\partial^2\Omega_i(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{jt}^2}=-1,\quad \frac{\partial^2\Omega_j(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{jt}^2}=1,\quad \frac{\partial^2\Omega_0(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{jt}^2}=0\end{aligned}$$

Without loss of generality, assume that $i=1$ and $j=2$. Then

$$\begin{aligned}\frac{\partial\Pi_1^D(\mathbf{x},t)}{\partial p_{1t}} &= 1-p_{1t}-\frac{(1-p_{2t})^2}{2}\\ &\quad+\left[\widetilde{\Pi}_1(x_1,x_2,t-1)-p_{1t}\right]-[\Pi_1(x_1,x_2-1,t-1)-\Pi_1(x_1,x_2,t-1)]\,p_{2t}\\ \frac{\partial\Pi_2(\mathbf{x},t)}{\partial p_{2t}} &= \frac{(2p_{1t}+1-p_{2t})(1-p_{2t})}{2}\\ &\quad+\left[\widetilde{\Pi}_2(x_1,x_2,t-1)-p_{2t}\right](1-p_{2t})-[\Pi_2(x_1-1,x_2,t-1)-\Pi_2(x_1,x_2,t-1)]\,p_{1t}\end{aligned}$$

Apparently, Π_i^D is unimodal as $\dfrac{\partial\Pi_1^D(\mathbf{x},t)}{\partial p_{1t}}$ can take the value 0 at at most one p_{1t}. For Π_2, the FOC can be written as

$$\frac{\partial\Pi_2(\mathbf{x},t)}{\partial p_{2t}} = \frac{3}{2}(1-p_{2t})^2+\left[\widetilde{\Pi}_2(x_1,x_2,t-1)+\frac{p_{1t}}{2}-1\right](1-p_{2t})-[\Pi_2(x_1-1,x_2,t-1)-\Pi_2(x_1,x_2,t-1)]\,p_{1t}$$

Note that $\Pi_2(x_1-1,x_2,t-1)-\Pi_2(x_1,x_2,t-1)\geq 0$, therefore, one root (if any) of the FOC will satisfy $p_{2t}>1$. Thus, Π_2 as a function of p_{2t} is unimodal on $[0,1]$. These prove the existence of pure strategy NE. □

Proof of Proposition 2: For stage 2 at time t, the seller with lower reservation price r_i will be chosen by the NYOP intermediary as the opaque product provider. Consider seller 1, suppose his rival seller 2 proposes a reservation price $r_{2t}\geq\widetilde{\Pi_1}(\mathbf{x},t-1)$. Then, seller 1 can practically take three sets of actions:

1. *give up* the opaque provider-ship by setting $r_{1t}=1$. In this case, seller 1's expected revenue can be expressed by

$$\begin{aligned}\Pi_1^{>r}(\mathbf{p}|\mathbf{x},t) = {} & [\,p_1+\Pi_1(x_1-1,x_2,t-1)\,]\,\Omega_1(\mathbf{r}_t,\mathbf{p}_t)+\Pi_1(x_1,x_2-1,t-1)\Omega_O(\mathbf{r}_t,\mathbf{p}_t)\\ & +\Pi_1(x_1,x_2-1,t-1)\Omega_2(\mathbf{r}_t,\mathbf{p}_t)+\Pi_1(x_1,x_2,t-1)\Omega_0(\mathbf{r}_t,\mathbf{p}_t);\end{aligned}$$

2. *match* the reservation price by setting $r_{1t}=r_{2t}$. In this case, each seller is the opaque provider with equal probability.

$$\begin{aligned}&\Pi_1^{=r}(\mathbf{p}|\mathbf{x},t)=[\,p_1+\Pi_1(x_1-1,x_2,t-1)\,]\,\Omega_1(\mathbf{r}_t,\mathbf{p}_t)\\ &+\left\{\frac{1}{2}\,[\,r+\Pi_1(x_1-1,x_2,t-1)\,]+\frac{1}{2}\Pi_1(x_1,x_2-1,t-1)\right\}\Omega_O(\mathbf{r}_t,\mathbf{p}_t)\\ &+\Pi_1(x_1,x_2-1,t-1)\Omega_2(\mathbf{r}_t,\mathbf{p}_t)+\Pi_1(x_1,x_2,t-1)\Omega_0(\mathbf{r}_t,\mathbf{p}_t);\end{aligned}$$

3. *become* the opaque provider by agreeing to a lower reservation price $r_{1t}=r_{2t}-\epsilon$. As $\epsilon\to 0$, the expected revenue is given by

$$\begin{aligned}\Pi_1^{<r}(\mathbf{p}|\mathbf{x},t) = {} & [\,p_1+\Pi_1(x_1-1,x_2,t-1)\,]\,\Omega_1(\mathbf{r}_t,\mathbf{p}_t)+[\,r+\Pi_1(x_1-1,x_2,t-1)\,]\,\Omega_O(\mathbf{r}_t,\mathbf{p}_t)\\ & +\Pi_1(x_1,x_2-1,t-1)\Omega_2(\mathbf{r}_t,\mathbf{p}_t)+\Pi_1(x_1,x_2,t-1)\Omega_0(\mathbf{r}_t,\mathbf{p}_t).\end{aligned}$$

Comparing the three strategies, it is not hard to verify that it is optimal for seller i to *become* opaque provider whenever $r_{2t}>\widetilde{\Pi_1}(\mathbf{x},t-1)$, *match* when $r_{2t}=\widetilde{\Pi_1}(\mathbf{x},t-1)$, and *give up* when $r_{2t}<\widetilde{\Pi_1}(\mathbf{x},t-1)$.

Without loss of generality, assume that $\widetilde{\Pi_1}(\mathbf{x},t-1)\leq\widetilde{\Pi_2}(\mathbf{x},t-1)$. Then, seller 2 will compete with seller 1 for the opaque provider-ship until $r_{1t}=r_{2t}=\widetilde{\Pi_2}(\mathbf{x},t-1)$. And seller 1 can gain the full provider-ship by lowering down his reservation price to $\widetilde{\Pi_2}(\mathbf{x},t-1)-\epsilon$ where $\epsilon\to 0$. Therefore, the minimum reservation price is $r^*(\mathbf{x},t)=\widetilde{\Pi_2}(\mathbf{x},t-1)$. Seller 1 will be the unique opaque provider unless $\widetilde{\Pi_1}(\mathbf{x},t-1)=\widetilde{\Pi_2}(\mathbf{x},t-1)$, in which case both sellers are the opaque provider with equal probability. □

Proof of Theorem 3: We aim to prove that Π_i is unimodal in p_{it} for $i=1,2$. Without loss of generality, assume that $r_t=\widetilde{\Pi_2}(\mathbf{x},t-1)<\widetilde{\Pi_1}(\mathbf{x},t-1)$, i.e., seller 2 is the opaque-product provider. Then,

$$\begin{aligned}\Pi_1(\mathbf{x},t) &= [p_{1t}+\Pi_1(x_1-1,x_2,t-1)]\,\Omega_1(\mathbf{r}_t,\mathbf{p}_t)+\Pi_1(x_1,x_2-1,t-1)\Omega_O(\mathbf{r}_t,\mathbf{p}_t)\\ &\quad+\Pi_1(x_1,x_2-1,t-1)\Omega_2(\mathbf{r}_t,\mathbf{p}_t)+\Pi_1(x_1,x_2,t-1)\Omega_0(\mathbf{r}_t,\mathbf{p}_t) \qquad \text{(A9a)}\\ \Pi_2(\mathbf{x},t) &= \Pi_2(x_1-1,x_2,t-1)\Omega_1(\mathbf{r}_t,\mathbf{p}_t)+\left[\widetilde{\Pi_2}(\mathbf{x},t-1)+\Pi_2(x_1,x_2-1,t-1)\right]\Omega_O(\mathbf{r}_t,\mathbf{p}_t)\\ &\quad+[p_{2t}+\Pi_2(x_1,x_2-1,t-1)]\,\Omega_2(\mathbf{r}_t,\mathbf{p}_t)+\Pi_2(x_1,x_2,t-1)\Omega_0(\mathbf{r}_t,\mathbf{p}_t) \qquad \text{(A9b)}\end{aligned}$$

Note that

$$-\frac{\partial\Omega_O(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_i}=\frac{\partial\Omega_1(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_i}+\frac{\partial\Omega_2(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_i}+\frac{\partial\Omega_0(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_i}$$

for $i=1,2$. Then,

$$\begin{aligned}\frac{\partial\Pi_1}{\partial p_{1t}} &= \Omega_1-\left(\widetilde{\Pi_1}-p_{1t}\right)\frac{\partial\Omega_1}{\partial p_{1t}}-\Delta\Pi_1\frac{\partial\Omega_0}{\partial p_{1t}}\\ \frac{\partial^2\Pi_1}{\partial p_{1t}^2} &= 2\frac{\partial\Omega_1}{\partial p_{1t}}-\left[\widetilde{\Pi_1}-p_{1t}\right]\frac{\partial^2\Omega_1}{\partial p_{1t}^2}-\Delta\Pi_1\frac{\partial^2\Omega_0}{\partial p_{1t}^2}.\\ \frac{\partial\Pi_2}{\partial p_{2t}} &= \Omega_2-\left(\widetilde{\Pi_2}-p_{2t}\right)\frac{\partial\Omega_2}{\partial p_{2t}}-\Delta\Pi_2\frac{\partial\Omega_0}{\partial p_{2t}}\\ \frac{\partial^2\Pi_2}{\partial p_{2t}^2} &= 2\frac{\partial\Omega_2}{\partial p_{2t}}-\left[\widetilde{\Pi_2}-p_{2t}\right]\frac{\partial^2\Omega_2}{\partial p_{2t}^2}-\Delta\Pi_2\frac{\partial^2\Omega_0}{\partial p_{2t}^2}.\end{aligned}$$

For the ease of presentation, we omit all the function variables, and use $\Delta\Pi_1$ and $\Delta\Pi_2$ to denote $\Pi_1(x_1,x_2-1,t-1)-\Pi_1(x_1,x_2,t-1)$ and $\Pi_2(x_1-1,x_2,t-1)-\Pi_2(x_1,x_2,t-1)$ respectively. The same notations apply to the rest of the proof.

We first prove that Π_1 is unimodal in p_{1t}. Under the uniform valuation assumption, $\mathbf{H}_{DC}(\mathbf{v})$ can be rewritten as follows:

$$\mathbf{H}_{DC}(\mathbf{v})=\begin{cases}(0,0) & \text{if } \max\{v_1-p_1,v_2-p_2\}<0 \text{ and } \alpha_1v_1+\alpha_2v_2\le 2r;\\ (1,p_1) & \text{if } v_1-p_1\ge\max\{v_2-p_2,0\} \text{ and } v_1-v_2\ge(p_1-2r)/\alpha_2;\\ (2,p_2) & \text{if } v_2-p_2\ge\max\{v_1-p_1,0\} \text{ and } v_2-v_1\ge(p_2-2r)/\alpha_1;\\ (O,b^*) & \text{otherwise.}\end{cases} \qquad \text{(A10)}$$

To avoid trivial cases, assume that $2r_t\le 1$. Depending on the value of p_{1t}, its first order effect on Π_1 can take the following forms:

I. When $\alpha_1 p_{1t} + \alpha_2 p_{2t} < 2r_t$ and $p_{1t} \le p_{2t}$, there are $\Omega_1 = 1 - \frac{2p_{1t} + (1-p_{2t})^2}{2}$ and $\Omega_0 = p_{1t}p_{2t}$. Hence,

$$\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}} = 1 - 2p_{1t} - \frac{(1-p_{2t})^2}{2} + \widetilde{\Pi_1} - \Delta\Pi_1 p_{2t} = -(p_{1t} - 1 - \Delta\Pi_1)^2 + \frac{(1+\Delta\Pi_1)^2 + 1}{2} + \widetilde{\Pi_1}$$

II. When $\alpha_1 p_{1t} + \alpha_2 p_{2t} < 2r_t$ and $p_{2t} \le p_{1t}$, there are $\Omega_1 = \frac{(2p_{2t} + 1 - p_{1t})(1-p_{1t})}{2}$ and $\Omega_0 = p_{1t}p_{2t}$. Thus,

$$\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}} = \frac{3}{2}(1-p_{1t})^2 + (2p_{2t} + \widetilde{\Pi_1} - 1)(1-p_{1t}) - p_{2t}(1 + \Delta\Pi_1 - \widetilde{\Pi_1})$$

III. When $\alpha_1 p_{1t} + \alpha_2 p_{2t} \ge 2r_t$ and $p_{1t} < 2r_t$, there are $\Omega_1 = -\frac{\alpha_1^2}{2\alpha_2^2}p_{1t}^2 - p_{1t}(\frac{1}{\alpha_2} - 2r_t\frac{\alpha_1}{\alpha_2^2}) + \frac{1}{2} - \frac{2r_t^2}{\alpha_2^2} + \frac{2r_t}{\alpha_2}$ an $\Omega_0 = -\frac{\alpha_1}{2\alpha_2}p_{1t}^2 - \frac{\alpha_2}{2\alpha_1}p_{2t}^2 + \frac{2r_t}{\alpha_2}p_{1t} + \frac{2r_t}{\alpha_1}p_{2t} - \frac{2r_t^2}{\alpha_1\alpha_2}$. Hence,

$$\begin{aligned}\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}} &= -\frac{3}{2}\frac{\alpha_1^2}{\alpha_2^2}p_{1t}^2 - [\frac{2}{\alpha_2} - 4\frac{r_t\alpha_1}{\alpha_2^2} - \frac{\alpha_1^2}{\alpha_2^2}\widetilde{\Pi_1} - \Delta\Pi_1\frac{\alpha_1}{\alpha_2}]p_{1t} \\ &\quad + \frac{1}{2} + 2\frac{r_t}{\alpha_2} - \frac{2r_t^2}{\alpha_2^2} + (\frac{1}{\alpha_2} - 2\frac{r_t\alpha_1}{\alpha_2^2})\widetilde{\Pi_1} - 2\frac{r_t}{\alpha_2}\Delta\Pi_1\end{aligned}$$

IV. When $\alpha_1 p_{1t} + \alpha_2 p_{2t} \ge 2r_t$ and $2r_t \le p_{1t} < 2\frac{r_t}{\alpha_1}$, there are $\Omega_1 = \frac{(1+2\frac{\alpha_1}{\alpha_2})p_{1t}^2 - (\frac{2}{\alpha_2} + 4\frac{r_t}{\alpha_2})p_{1t} + 1 + 4\frac{r_t}{\alpha_2}}{2}$ and $\Omega_0 = -\frac{\alpha_1}{2\alpha_2}p_{1t}^2 - \frac{\alpha_2}{2\alpha_1}p_{2t}^2 + \frac{2r_t}{\alpha_2}p_{1t} + \frac{2r_t}{\alpha_1}p_{2t} - \frac{2r_t^2}{\alpha_1\alpha_2}$. Hence,

$$\begin{aligned}\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}} &= \frac{3}{2}(1+2\frac{\alpha_1}{\alpha_2})p_{1t}^2 - [\frac{2}{\alpha_2} + 4\frac{r_t}{\alpha_2} + (1+2\frac{\alpha_1}{\alpha_2})\widetilde{\Pi_1} - \Delta\Pi_1\frac{\alpha_1}{\alpha_2}]p_{1t} \\ &\quad + \frac{1}{2} + 2\frac{r_t}{\alpha_2} + (2\frac{r_t}{\alpha_2} + \frac{1}{\alpha_2})\widetilde{\Pi_1} - 2\frac{r_t}{\alpha_2}\Delta\Pi_1\end{aligned}$$

V. When $2\frac{r_t}{\alpha_1} \le p_{1t} < \alpha_2 + 2r_t$, there are $\Omega_1 = \frac{(1 - \frac{p_{1t}-2r_t}{\alpha_2})^2}{2}$ and Ω_0 is independent of p_{1t}. Thus,

$$\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}} = \frac{(1 - \frac{p_{1t}-2r_t}{\alpha_2})^2}{2} + \frac{1}{\alpha_2}(\widetilde{\Pi_1} - p_{1t})(1 - \frac{p_{1t}-2r_t}{\alpha_2})$$

VI. When $\alpha_2 + 2r_t \le p_{1t} \le 1$, there are $\Omega_1 = 0$ and Ω_0 independent of p_{1t}. Thus, $\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}} = 0$.

We next prove the unimodal by showing that there exists an $p_{1t}^* \in [0,1]$ such that $\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}} \ge 0$ if $0 \le p_{1t} \le p_{1t}^*$, and $\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}} \le 0$ if $p_{1t}^* \le p_{1t} \le 1$.

First note that $\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}$ is continuous in Scenario I and II. In Scenario I, it increases in p_{1t} and

$$\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}|_{p_{1t}=0} = \frac{1}{2} - \frac{p_{2t}^2}{2} + (1-\Delta\Pi_1)p_{2t} + \widetilde{\Pi_1} > 0.$$

So $\dfrac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}} > 0$ in this scenario.

In Scenario II, $\dfrac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}$ is convex. Since $-p_{2t}(1+\Delta\Pi_1 - \widetilde{\Pi_1}) < 0$, at least one (if any) of the roots of the first-order condition (FOC) should be greater than 1. Also, as

$$\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}|_{p_{1t}=0} = \frac{1}{2} + 4r_t + (4r_t+2)\widetilde{\Pi_1} - 4r_t\Delta\Pi_1 > 0,$$

there is one $p_{1t} \in [0,1]$ at which the FOC is achieved, denoted as p_{1t}^{II}.

In Scenario III, $\dfrac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}$ is concave and

$$\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}|_{p_{1t}=0} = \frac{1}{2} + 2\frac{r_t}{\alpha_2} - \frac{2r_t^2}{\alpha_2^2} + (-2\frac{r_t\alpha_1}{\alpha_2} + 1 + \frac{\alpha_1}{\alpha_2})\widetilde{\Pi_1} - 2\frac{r_t}{\alpha_2}\Delta\Pi_1 > 0$$

Then exactly one root of FOC is positive, which is denote as p_{1t}^{III}.

In Scenario IV, $\dfrac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}$ is convex and

$$\begin{aligned}\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}|_{p_{1t}=0} &= \frac{1}{2} + 4r_t + (4r_t+2)\widetilde{\Pi_1} - 4r_t\Delta\Pi_1 > 0\\ \frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}|_{p_{1t}=1} &= (1-4r_t)(1-\widetilde{\Pi_1} - \Delta\Pi_1) < 0\end{aligned}$$

Thus, there is one $p_{1t} \in [0,1]$ at which FOC is achieved, denoted as p_{1t}^{IV}.

In scenario V, the FOC can be achieved at $p_{1t} = \dfrac{\alpha_2 + 2r_t + 2\widetilde{\Pi_1}}{3}$ and $p_{1t} = \alpha_2 + 2r_t$, where $\dfrac{\alpha_2 + 2r_t + 2\widetilde{\Pi_1}}{3} \le \alpha_2 + 2r_t$. Denote $p_{1t}^V = \dfrac{\alpha_2 + 2r_t + 2\widetilde{\Pi_1}}{3}$. If $p_{1t}^V \le \frac{2r_t}{\alpha_1}$, then $\dfrac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}$ is constantly negative in Scenario V. Otherwise, it can be shown that $\dfrac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}$ in Scenario IV will be 0 at $p_{1t} = \frac{2r_t}{\alpha_1}$, i.e., $p_{1t}^{IV} = \frac{2r_t}{\alpha_1}$.

Thus, p_{1t}^* can only be among p_{1t}^{II}, p_{1t}^{III}, p_{1t}^{IV} and p_{1t}^V. It can be shown that p_{1t}^{III} and p_{1t}^{IV} cannot both be feasible at the same time, i.e., $p_{1t}^{II} \le 2r_t \le p_{1t}^{IV}$ cannot hold true. Otherwise, it can be verified that

$$\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}^{III}|_{p_{1t}=2r_t} - \frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}^{IV}|_{p_{1t}=2r_t} \sim 1 + \alpha_1^2 - \alpha_2^2 \ge 0$$

However, for what we have discussed about $\dfrac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}$, there should be $\dfrac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}^{IV}|_{p_{1t}=2r_t} \ge \dfrac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}^{IV}|_{p_{1t}=p_{1t}^{IV}} = 0 = \dfrac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}^{III}|_{p_{1t}=p_{1t}^{III}} \ge \dfrac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}^{III}|_{p_{1t}=2r_t}$. The same reasoning applies to p_{1t}^{II} vs. p_{1t}^{III} and p_{1t}^{IV} vs. p_{1t}^V .

Therefore, it can be concluded that at most one of p_{1t}^{II}, p_{1t}^{III}, p_{1t}^{IV} and p_{1t}^V will be feasible and $\dfrac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}} \ge 0$ (resp. ≤ 0) when p_{1t} is less (resp. greater) than it. If none of the p_{1t}^{II}, p_{1t}^{III}, p_{1t}^{IV} or p_{1t}^V is feasible, the profit function is monotone on $[0,1]$ and $\Pi_1(\mathbf{x},t)$ is still unimodal in p_{1t}.

The proof for $\Pi_2(\mathbf{x},t)$ can be done in a similar fashion. Therefore, there exists a pure NE for the posted prices (p_{1t}, p_{2t}). □

Proof of Corollary 2:

(i) At $t=1$, the marginal value of inventory $\widetilde{\Pi}_i(\mathbf{x},t-1)$ is zero for either seller. By Proposition 2, the reservation price is $r^*=\max_i \widetilde{\Pi}_i(\mathbf{x},t-1)=0$.

(ii) We can show that, at any time t, if $x_1 \geq t$ and $x_2 \geq t$,

$$\Pi_1(x_1,x_2,t)=\Pi_2(x_1,x_2,t)=\pi_t \tag{A11}$$

for some $\pi_t>0$. Hence $\widetilde{\Pi}_i(\mathbf{x},t)=0$ for any $x_1\geq t$ and $x_2 \geq t$, which immediately leads to $r^*=0$.

We show (A11) by induction. First, it is apparent that (A11) holds for $t=1$. Now, suppose (A11) holds for all $t<T$. We then have $\Pi_1(x_1-1,x_2,T-1)=\Pi_1(x_1-1,x_2-1,T-1)=\Pi_2(x_1,x_2-1,T-1)=\Pi_2(x_1-1,x_2-1,T-1)$ for any $x_1\geq T$ and $x_2\geq T$. Thus, $\widetilde{\Pi}_i(x_1,x_2,T-1)=0$ for $i=1,2$ and

$$r^*(x_1,x_2,T)=0, \qquad \forall x_1\geq T, x_2\geq T. \tag{A12}$$

(A12) implies that, when both sellers oversupply, the price of the opaque goods will remain zero until one seller's inventory becomes lower than the potential demand. The expected profit for seller 1 is then

$$\begin{aligned}\Pi_1(\mathbf{x},T) &= [\,p_{1T}+\Pi_1(x_1-1,x_2,T-1)\,]\,\Omega_1(\mathbf{r}_t,\mathbf{p}_t)+\Pi_1(x_1-1,x_2,T-1)\Omega_O(\mathbf{r}_t,\mathbf{p}_t)\\ &\quad +\Pi_1(x_1,x_2-1,T-1)\Omega_2(\mathbf{r}_t,\mathbf{p}_t)+\Pi_1(x_1,x_2,T-1)\Omega_0(\mathbf{r}_t,\mathbf{p}_t)\\ &= p_{1T}\Omega_1(\mathbf{r}_t,\mathbf{p}_t)+\pi_{T-1},\end{aligned} \tag{A13}$$

which does not depend on $\mathbf{x}$ for $x_1\geq T$ and $x_2\geq T$. Similarly, $\Pi_2(\mathbf{x},T)=p_{2T}\Omega_2(\mathbf{r}_t,\mathbf{p}_t)+\pi_{T-1}$. When $r=0$, $\Omega_i(\mathbf{r}_t,\mathbf{p}_t)=\dfrac{(1-p_{it}/\alpha_i)^2}{2}$ for $i=1,2$. It is then straightforward that in equilibrium there should be $p_{1T}=p_{2T}$ and $\Pi_1(\mathbf{x},T)=\Pi_2(\mathbf{x},T)$. Thus, (A11) also holds for $t=T$. This completes the proof. $\square$

Proof of Proposition 3: Apparently there is oversupply when $x_1=x_2=\infty>T$. For a similar analysis as in the proof of Corollary 2 (ii), under DC the reservation price is $r_i^*=0$ and expected profit $\Pi_i^{DC}(x_1,x_2,t)=\pi_t^{DC}$; under SC the expected profit $\Pi_i^{SC}(x_1,x_2,t)=\pi_t^{SC}$, for $i=1,2$ and any t;

Specifically for $T=1$, the optimal posted price under DC is set by $p_{iT}^{DC}=\arg\max_{0\leq p_{iT}\leq 1} p_{iT}\frac{(1-p_{iT}/\alpha_i)^2}{2}=\frac{\alpha_i}{3}$ and $\Pi_{iT}^{DC}=\frac{2\alpha_i}{27}$. However, under SC the equilibrium posted price is $p_{iT}^{SC}=\sqrt{2}-1$ and $\Pi_{iT}^{SC}=3-2\sqrt{2}>2/27>\Pi_{iT}^{DC}=2\alpha_i/27$. Therefore at $T=1$, the equilibrium channel structure is SC.

Suppose SC is the equilibrium channel structure for all $T<t$. At $T=t$ under DC, by (A13) the expected profit for seller i is $\Pi_{it}^{DC}=2\alpha_i/27+\pi_{t-1}^{DC}$. At $T=t$ under SC, the expected profit for seller i is $\Pi_{it}^{SC}=3-2\sqrt{2}+\pi_{t-1}^{SC}$. Since SC is the equilibrium at $t-1$, there should be $\pi_{t-1}^{SC}\geq\pi_{t-1}^{DC}$. Thus $\Pi_{it}^{SC}>\Pi_{it}^{DC}$. Hence SC is the equilibrium at $T=t$.

This proves the claim that SC is the equilibrium when supply is unlimited. □

Proof of Proposition 4: To prove that SDCi is an equilibrium when $x_1\to\infty$ and $x_2\to\infty$, it is suffice to show that $\Pi_1^{10}(\infty,\infty,T)\geq\Pi_1^{00}(\infty,\infty,T)$ and $\Pi_1^{01}(\infty,\infty,T)\geq\Pi_1^{11}(\infty,\infty,T)$ for any $T>0$.

At $T=1$, it can be verified that reservation price will be set high under SDCi structure and all transactions will be realized via the direction channel. By the proof of Proposition 3 there are $\Pi_1^{10}(\infty,\infty,1)=\Pi_1^{00}(\infty,\infty,1)=3-2\sqrt{2}$ and $\Pi_1^{01}(\infty,\infty,1)=3-2\sqrt{2}\geq\Pi_1^{11}(\infty,\infty,1)=2\alpha_i/27$. Thus SDC$i$ is an NE when $T=1$.

Suppose the statements hold true for $T<t$. Then at $T=t$, by Corollary 2 (ii) the reservation price under DC is 0. Therefore.

$$\begin{aligned}
\Pi_1^{11}(\infty,\infty,t)= &\quad \frac{1}{2}\Omega_O(\mathbf{r}_t,\mathbf{p}_t)\Pi_1^{11}(\infty,\infty,t-1)+\Omega_1(\mathbf{r}_t,\mathbf{p}_t)\left[p_{1t}+\Pi_1^{11}(\infty,\infty,t-1)\right]\\
&\quad +[\Omega_2(\mathbf{r}_t,\mathbf{p}_t)+\frac{1}{2}\Omega_O(\mathbf{r}_t,\mathbf{p}_t)]\Pi_1^{11}(\infty,\infty,t-1)+\Omega_0(\mathbf{r}_t,\mathbf{p}_t)\Pi_1^{11}(\infty,\infty,t-1)\\
&=\Pi_1^{11}(\infty,\infty,t-1)+\Omega_1(\mathbf{0},\mathbf{p}_t)p_{1t}
\end{aligned}$$

Thus in the equilibrium of DC, there will be $\Pi_1^{11}(\infty,\infty,t)=\Pi_1^{11}(\infty,\infty,t-1)+\Pi_1^{11}(\infty,\infty,1)$.

Similarly for SDC2,

$$\begin{aligned}
\Pi_1^{01}(\infty,\infty,t)= &\quad \Omega_1(\mathbf{r}_t,\mathbf{p}_t)\left[p_{1t}+\Pi_1^{01}(\infty,\infty,t-1)\right]+[\Omega_2(\mathbf{r}_t,\mathbf{p}_t)+\Omega_O(\mathbf{r}_t,\mathbf{p}_t)]\Pi_1^{01}(\infty,\infty,t-1)\\
&\quad +\Omega_0(\mathbf{r}_t,\mathbf{p}_t)\Pi_1^{01}(\infty,\infty,t-1)\\
&=\Pi_1^{01}(\infty,\infty,t-1)+\Omega_1(\mathbf{r}_t,\mathbf{p}_t)p_{1t}
\end{aligned}$$

and at equilibrium there is $\Pi_1^{01}(\infty,\infty,t)=\Pi_1^{01}(\infty,\infty,t-1)+\Pi_1^{01}(\infty,\infty,1)$.

Since $\Pi_1^{01}(\infty,\infty,T)\geq\Pi_1^{11}(\infty,\infty,T)$ for any $T<t$, there should be $\Pi_1^{01}(\infty,\infty,t)=\Pi_1^{01}(\infty,\infty,t-1)+\Pi_1^{01}(\infty,\infty,1)\geq\Pi_1^{11}(\infty,\infty,t-1)+\Pi_1^{11}(\infty,\infty,1)=\Pi_1^{11}(\infty,\infty,t)$. Thus $\Pi_1^{01}(\infty,\infty,T)\geq\Pi_1^{11}(\infty,\infty,T)$ also holds for $T=t$. The same argument applies to $\Pi_1^{10}(\infty,\infty,t)\geq\Pi_1^{00}(\infty,\infty,t)$. Therefore, the statements also hold for $T=t$. This proves that in general SDCi is an equilibrium when inventory is unlimited. □

Proof of Proposition 5:

(i) For vertical differentiation, consider that $\{\mathbf{v}_t:v_{it}-v_{jt}=\bar{v}_t\}$ for some constant $\bar{v}_t>0$. By $\mathbf{H}_{DC}(\mathbf{v})$

in §4.3, a customer will consistently buy from the *same* type of channel (i.e., from seller i, or seller j, or the NYOP channel) or leave empty handed. (*)

• Consider $t=1$, the last period sales. Due to Theorem 1, the statement apparently holds when one seller is out of stock. Now, suppose both sellers are in stock. Corollary 2 implies that both sellers has marginal inventory value zero thus $r_1=0$ hence no seller earns from the NYOP channel. By (*), sellers' should set posted prices to induce the customer purchase through direct channel ultimately.

We first argue that a customer will not buy directly from seller j. First, there is always a strategy in which seller i can set his posted prices as $p_{i1}=p_{j1}+v_{i1}-v_{j1}-\varepsilon=p_{j1}+\bar{v}_t-\varepsilon$ for some $\varepsilon>0$ such that the last-period customer will always prefer to buy from seller i at p_{i1}. Second, it is optimal for seller i to set such a posted price in winning the potential customer, since direct channel is the only place that generates income. The competition then drives the period-1 posted prices for i and j to $p_{i1}=\bar{v}_1-\varepsilon$ and $p_{j1}=0$ respectively.

Next, to ensure that the customer will prefer buying directly from seller i than obtaining opaque product from NYOP channel, the posted price for i should also satisfy $\alpha_i v_{i1}+\alpha_j v_{j1}\le v_{i1}-p_{i1}$, hence $p_{i1}\le\alpha_j\bar{v}_1$. The equilibrium at $t=1$ is therefore $p_{i1}=\alpha_j\bar{v}_1$ and $p_{j1}=0$. The expected profit for seller i is $\alpha_j\bar{v}_1$. seller j and the NYOP firm earns zero.

The statement is true for $t=1$.

• Now consider $t>1$. Denote $v_{it}^r=r_t^*+\dfrac{G(r_t^*)}{g(r_t^*)}+\alpha_j\bar{v}_t$ and $v_{jt}^r=r_t^*+\dfrac{G(r_t^*)}{g(r_t^*)}-\alpha_i\bar{v}_t$. There is $v_{it}^r-v_{jt}^r=\bar{v}_t$. Then, if $v_{it}<p_{it}$ and $v_{jt}<p_{jt}$, customers with $v_{it}\ge v_{it}^r$ or $v_{jt}\ge v_{jt}^r$ will bid above the reservation price r_t^* and purchase through the NYOP channel. Other customers cannot afford either direct channel and will leave empty handed.

If $v_{it}\ge p_{it}$ and $\bar{v}_t\ge p_{it}-p_{jt}$, analysis in §4.3 suggests that the optimal bid satisfies $\min\{v_{it},p_{it}\}=b^*+\dfrac{G(b^*)}{g(b^*)}+\alpha_j\bar{v}_t$. Thus those with $\min\{v_{it},p_{it}\}<v_{it}^r$ will bid below r_t^* and will purchase through direct channel i in the end.

If $v_{jt}\ge p_{jt}$ and $\bar{v}_t\le p_{it}-p_{jt}$, the optimal bid satisfies $\min\{v_{jt},p_{jt}\}=b^*+\dfrac{G(b^*)}{g(b^*)}-\alpha_i\bar{v}_t$, and those with $\min\{v_{jt},p_{jt}\}<v_{jt}^r$ will bid below r_t^* and will purchase through direct channel j in the end.

With a slight abuse of the notation, denote $\widetilde{\Pi}_{it}=\widetilde{\Pi}_i(\mathbf{x},t)$ and $\widetilde{\Pi}_{jt}=\widetilde{\Pi}_j(\mathbf{x},t)$. We next characterize the equilibrium pricing strategy when seller i is the opaque seller (i.e., $\widetilde{\Pi}_{it}\le\widetilde{\Pi}_{jt}$). For seller i's optimal pricing response given p_{jt}:

• First consider the case when $p_{jt}\ge v_{jt}^r$. If $p_{it}\ge v_{it}^r$, customers with $v_i\ge v_{it}^r$ will purchase through the NYOP channel, and those with $v_i<v_{it}^r$ are not accepted by the NYOP channel and cannot afford either direct channel, hence will left empty-handed. Seller i's expected profit is $\Pi_1(\mathbf{x},t)=\Pi_1(x_1-1,x_2,t-1)\dfrac{1-v_{it}^r}{1-\bar{v}_t}+\Pi_1(x_1,x_2,t-1)\dfrac{v_{it}^r-\bar{v}_t}{1-\bar{v}_t}$. If $p_{it}<v_{it}^r$, bids from all customers will be

rejected. The only realized sales are through direct channel i. The expected profit for seller i is $\Pi_1(\mathbf{x},t)=[\,p_{it}+\Pi_1(x_1-1,x_2,t-1)\,]\,\dfrac{1-p_{it}}{1-\bar{v}_t}+\Pi_1(x_1,x_2,t-1)\dfrac{p_{it}-\bar{v}_t}{1-\bar{v}_t}$. Apparently, it is optimal for seller i to set $p_{it}=v_{it}^r-\epsilon$ for some small $\epsilon>0$ in this scenario.

- For the case where $p_{jt}<v_{jt}^r$, if $p_{it}>p_{jt}+\bar{v}_t$, then all bids will be rejected and customers with $v_{jt}\ge p_{jt}$ will buy through direct channel j in the end. Seller i's expected profit is $\Pi_1(\mathbf{x},t)=\Pi_1(x_1,x_2-1,t-1)\dfrac{1-p_{jt}-\bar{v}_t}{1-\bar{v}_t}+\Pi_1(x_1,x_2,t-1)\dfrac{p_{jt}}{1-\bar{v}_t}$. If $p_{it}<p_{jt}+\bar{v}_t$, all bids will be rejected and customers with $v_{it}\ge p_{it}$ will buy through direct channel i in the end. Seller i's expected profit is $\Pi_1(\mathbf{x},t)=[\,p_{it}+\Pi_1(x_1-1,x_2,t-1)\,]\,\dfrac{1-p_{it}}{1-\bar{v}_t}+\Pi_1(x_1,x_2,t-1)\dfrac{p_{it}-\bar{v}_t}{1-\bar{v}_t}$. Comparing the two options, it is optimal for seller i to set $p_{it}=p_{jt}+\bar{v}_t-\epsilon$ for some small $\epsilon>0$ if $p_{jt}+\bar{v}_t>\widetilde{\Pi}_{it}$.

Overall, $p_{it}^*(p_{jt})=\min\{\max\{\widetilde{\Pi}_{it},p_{jt}+\bar{v}_t\},v_{it}^r\}$. Similarly for seller j, it can be verified that $p_{jt}^*(p_{it})=\min\{\max\{\widetilde{\Pi}_{jt},p_{it}-\bar{v}_t\},v_{jt}^r\}$. The same pair of response functions hold when seller j is the opaque product provider (i.e., $\widetilde{\Pi}_{it}\ge\widetilde{\Pi}_{jt}$). In either case, sales are realized through one direct channel only, and no sales will be through the NYOP channel.

(ii) For the horizontal differentiation case, consider that $\{\alpha_i v_i+\alpha_j v_j=\bar{v}_t\}$ for some $\bar{v}_t>0$. $\mathbf{H}_{DC}(\mathbf{v})$ suggests that there are *multiple* types of channel a customer can possibly end up with, depending on her valuation realization (v_{it},v_{jt}).

Denote $v_t^*=r_t^*+\dfrac{G(r_t^*)}{g(r_t^*)}$, where $r_t^*=\min\{\widetilde{\Pi}_{it},\widetilde{\Pi}_{jt}\}$ is the reservation price in period t. Then, if $\bar{v}_t\ge v_t^*$, all customers will bid above r_t^* when $v_{it}-p_{it}<0$ and $v_{jt}-p_{jt}<0$. We next show that if $\bar{v}_t\ge v_t^*$, then there exists some conditions under which $\Omega_O>0$ with equilibrium pricing (p_{it}^*,p_{jt}^*).

We start with analyzing seller i's optimal pricing response given p_{jt}:

- If $p_{it}\le v_t^r-p_{jt}$, then $\Omega_i=\alpha_j\dfrac{v_t^r-\alpha_i p_{it}+\alpha_i p_{jt}}{v_t^r}$ and $\Omega_0=\Omega_O=0$. As analyzed in the proof of Theorem 3,

$$\frac{\partial\Pi_i}{\partial p_{it}}=\Omega_i-\left(\widetilde{\Pi_i}-p_{it}\right)\frac{\partial\Omega_i}{\partial p_{it}}-\Delta\Pi_i\frac{\partial\Omega_0}{\partial p_{it}}\tag{A14}$$

where $\Delta\Pi_i=\Pi_i(x_i,x_j-1,t-1)-\Pi_i(x_i,x_j,t-1)$. Then,

$$\begin{aligned}\frac{\partial\Pi_i}{\partial p_{it}}&=\alpha_j\frac{v_t^r-\alpha_i p_{it}+\alpha_i p_{jt}}{v_t^r}-\left(p_{it}-\widetilde{\Pi_i}\right)\frac{\alpha_i\alpha_j}{v_t^r}=\frac{\alpha_i\alpha_j}{v_t^r}\left(\frac{v_t^r}{\alpha_i}-2p_{it}+p_{jt}+\widetilde{\Pi}_{it}\right)\\&\ge\frac{\alpha_i\alpha_j}{v_t^r}\left(\frac{v_t^r}{\alpha_i}-2v_t^r+3p_{jt}+\widetilde{\Pi}_{it}\right)\end{aligned}$$

- If $p_{it}>v_t^r-p_{jt}$, then $\Omega_i=\dfrac{v_t^r-\alpha_i p_{it}}{v_t^r}$, $\Omega_j=\dfrac{v_t^r-\alpha_j p_{jt}}{v_t^r}$, $\Omega_O=\dfrac{\alpha_i p_{it}+\alpha_j p_{jt}-v_t^r}{v_t^r}$ and $\Omega_0=0$. Thus,

$$\frac{\partial\Pi_i}{\partial p_{it}}=\frac{v_t^r-\alpha_i p_{it}}{v_t^r}-\left(p_{it}-\widetilde{\Pi_i}\right)\frac{\alpha_i}{v_t^r}=\frac{\alpha_i}{v_t^r}\left(\frac{v_t^r}{\alpha_i}-2p_{it}+\widetilde{\Pi}_{it}\right)$$

The first order derivative is greater than zero if $p_{it}<\dfrac{\frac{v_t^r}{\alpha_i}+\widetilde{\Pi}_{it}}{2}$.

The same analysis can be done for seller j. It can be verified that the equilibrium is

$$(p_{it}^*, p_{jt}^*) = \left(\frac{\dfrac{v_t^r}{\alpha_i} + \widetilde{\Pi}_{it}}{2}, \frac{\dfrac{v_t^r}{\alpha_j} + \widetilde{\Pi}_{jt}}{2} \right),$$

at which $\Omega_O = \dfrac{\alpha_i \widetilde{\Pi}_{it} + \alpha_j \widetilde{\Pi}_{jt}}{2v_t^r} > 0$ if one of $\widetilde{\Pi}_{it}$ and $\widetilde{\Pi}_{jt}$ is nonzero. Thus the NYOP channel earns non-zero profit when $v_t^r \leq \bar{v}_t$ and $\max\{\widetilde{\Pi}_{it}, \widetilde{\Pi}_{jt}\} > 0$.

□